HYDROSTATIC FORCES ON SURFACES

(I) Hydrostatic forces deals with the fluids at rest.

(II) There will be no relative motion b/w adjacent or neighbouring fluid layers.

(III). The velocity gradient, which is equal to the velocity b/w two adjacent fluid layers divided by the distance b/w the layers will be zero.

i.e. $\frac{du}{dy} = 0$.

(IV) The shear stress $\tau = \mu \cdot \frac{du}{dy} = 0$.

(V). The forces acting on the fluid particles will be

(1) pressure of fluid normal to the surface.

(2.) due to gravity or self weight of fluid particles.

TOTAL PRESSURE:

The total pressure is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surfaces.

The force always act normal to the surface.

CENTER OF PRESSURE:

Center of pressure is defined as the point of application of total pressure on the surface.

There are four cases of submerged surfaces on which the total pressure force and center of pressure is to be determined.

The submerged surfaces are: (I) Vertical plane surface

(II) Horizontal plane surface

(III) Curved surface

(IV) Inclined plane surface.

$$q = \frac{2}{3} C_d L \sqrt{2g}\, H^{3/2} + \frac{8}{15} C_d \tan(\theta/2)\sqrt{2g}\, H$$

VENTURIMETER $q = C_d\sqrt{2gh}\ (A_1 A_2)$

VERTICAL PLANE SURFACE SUBMERGED IN LIQUID:

A = Total area of the surface.

$\bar{h}$ = Distance of C.G. of the area from free surface of liquid.

G = center of gravity of plane surface.

P = center of pressure

h^* = distance of center of pressure from free surface of liquid.

TOTAL PRESSURE FORCE (F):

pressure intensity on the strip.

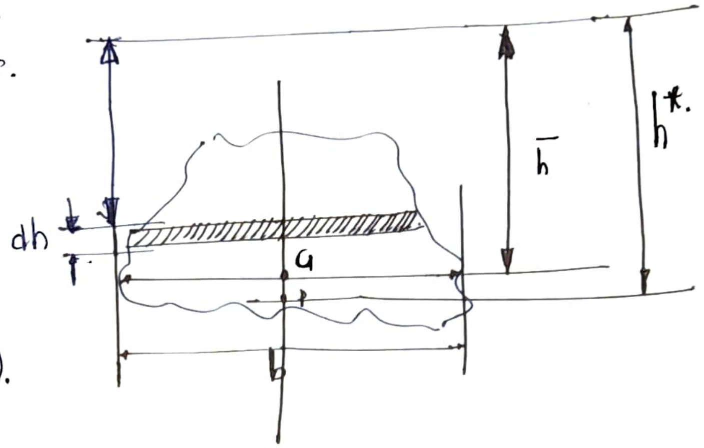
$p = \rho g h$.

Area of the strip.

$dA = b \times dh$

Total Force on the strip.

$dF = p \times dA$.



$= \rho g h \times (b \times dh)$.

$$\int dF = \rho g \int b h \times dh$$

$F = \rho g \int h\, dA$.

Total pressure force on the whole surface:

$$\boxed{F = \rho g \bar{h} A}$$

$\int h\,dA$ = Moment of surface area about free surface of liquid

= Area of surface × distance of C.G. from free surface

For water $\rho_w = 1000\ kg/m^3$.

CENTER OF PRESSURE (h^*)

center of pressure is calculated by using the "principle of moments" which states that the moment of resultant force about an axis is equal to the sum of the moments of the components about the same axis.

The resultant force F acting at P at a distance h^* from the free surface of the liquid.

Moment of force $dM = dF \times h$.

$= \rho g h (b \times dh) \times h = \rho g b \times h^2 dh$.

$$= h^2 \times (b \times dh)$$

$$M = \rho g \int h^2 \times dA.$$

$$M = \rho g \, I_0.$$

(Moment about free surface)

$$M = F \times h^*$$

$$F = \rho g \bar{h} A.$$

$$I_0 = I_G + A\bar{h}^2.$$

I_0 = Moment of inertia of the free surface about the free surface of the liquid.

By parallel axis Theorem.

$$I_0 = I_G + A\bar{h}$$

I_G = moment of inertia of area about an axis passing through the C.G. of the area and parallel to the free surface of the liquid.

$$(\rho g \bar{h} A) \times h^* = \rho g (I_G + A\bar{h}^2).$$

$$h^* = \frac{I_G + A\bar{h}^2}{A\bar{h}}$$

$$= \frac{I_G}{A\bar{h}} + \bar{h}$$

$$\boxed{h^* = \frac{I_G}{A\bar{h}} + \bar{h}}$$

$\bar{h}$ = distance of c.G. of the area of the vertical surface from free surface of the liquid.

(i) center of pressure (h^*) lies below the center of gravity of the vertical surface.

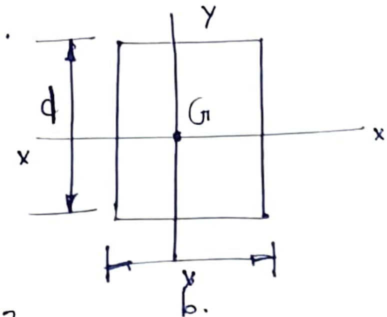
(ii) The distance of centere of pressure from the free surface of liquid is independent of the density of the liquid.

$$q = \frac{2}{3} C_d L\sqrt{2g}\, H^{3/2} + \frac{8}{15} C_d \tan(\theta/2)\sqrt{2g}\, H^{5/2}.$$

VENTURIMETER $Q = C_d\sqrt{2gh}\ (A_1 A_2)$

M.O.I

1.

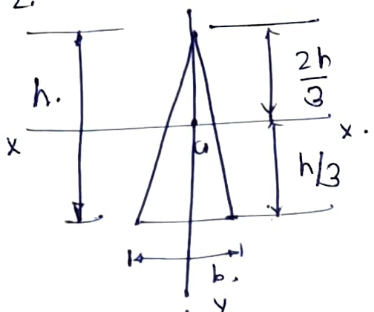


$I_{xx} = \frac{bd^3}{12}$

$I_{yy} = \frac{db^3}{12}.$

$A = b \times d.$

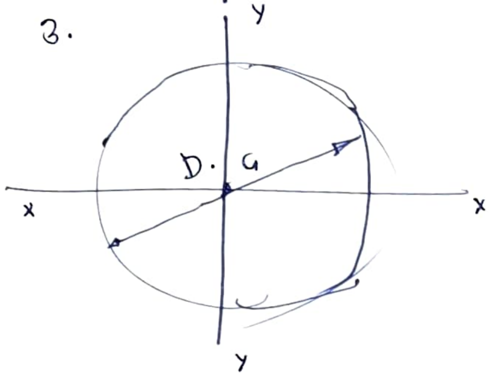
2.



$A = \frac{1}{2} b \times h.$

$I_{xx} = \frac{bh^3}{36}.$

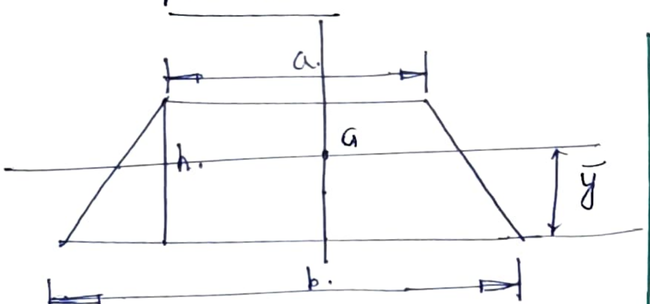
3.



$A = \frac{\pi}{4} \times D^2$

$I_{xx} = I_{yy} = \frac{\pi D^4}{64}.$

4. TRAPEZIUM



$$\boxed{\begin{aligned} \bar{y} &= \left(\frac{2a+b}{a+b}\right) \times \frac{h}{3}. \\ A &= \frac{1}{2}(a+b) \times h. \\ I_G &= \left[\frac{a^2 + 4ab + b^2}{36(a+b)}\right] \times h^3. \end{aligned}}$$

1. A rectangular plane surface is 2m wide and 4m deep. It lies in vertical plane in water. Determine the total pressure and position of center of pressure on the plane surface when its upper edge is horizontal and

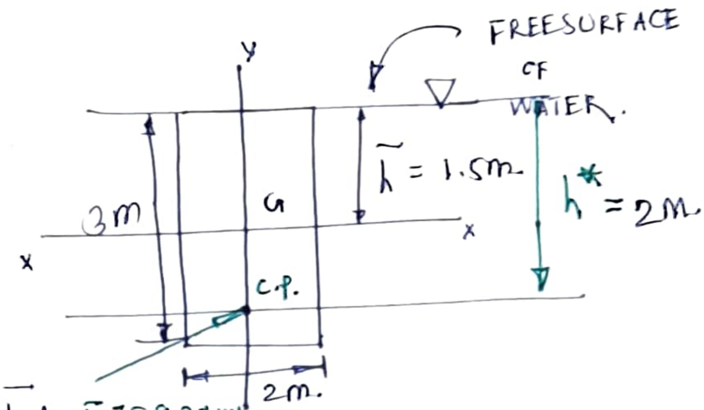
(I) conincides with water surface.

(II) 2.5m below the free water surface.

(I). $\rho = 1000 kg/m^3$

$A = 2 \times 3 = 6 m^2$

$\bar{h} = \frac{3}{2} = 1.5m.$



Total pressure Force: $F = \rho g \bar{h} A$ $F = 88.29 KN$

$= 1000 \times 9.81 \times 1.5 \times 6 = 88290N.$

$= 88.29 KN.$

$\boxed{F = 88.29 KN.}$

Depth of center of pressure: $h^* = \frac{I_G}{A\bar{h}} + \bar{h}$

$= \frac{4.5}{1.5 \times 6} + 1.5$

$= 0.5 + 1.5$

$= 2m$

$\boxed{h^* = 2m}$

I_G = M.O.I about X-X

$= \frac{b \times d^3}{12}$

$= \frac{2 \times 3^3}{12} = 4.5m^4.$

$\boxed{I_G = 4.5m^4}$

(II). $\bar{h} = 2.5 + \frac{3}{2} = 2.5 + 1.5$

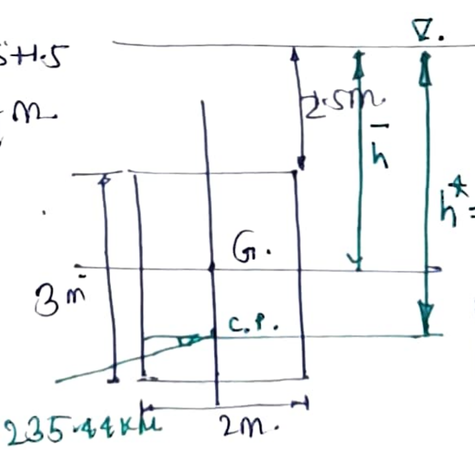
$= 4m.$

$F = 1000 \times 9.81 \times 4 \times 6$

$= 235440N$

$= 235.44 KN.$

$\boxed{F = 235.44 KN.}$



$h^* = \frac{4.5}{6 \times 4} + 4$

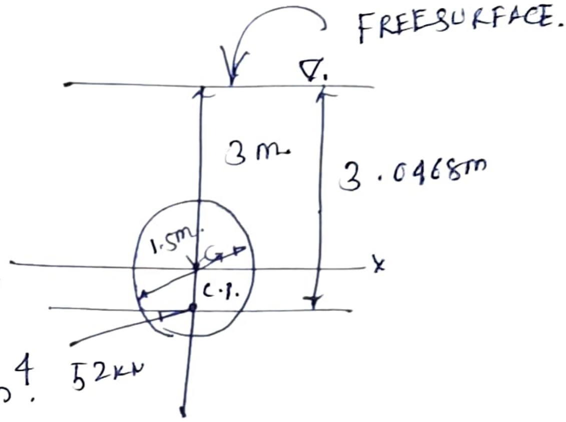
$= 4.1875m.$

$\boxed{h^* = 4.1875m.}$

$$q = \frac{2}{3} C_d \, L\sqrt{2g}\, H^{3/2} + \frac{8}{15} C_d \tan(\theta/2)\sqrt{2g}\, H$$

VENTURIMETER $\sqrt{2gh}$ $(A_1 A_2)$

2. Determine the total pressure on a circular plate of diameter 1.5 m which is placed vertically in water in such a way that the centre of the plate is 3 m below the free surface of water. Find the position of centre of pressure.



$\bar{h} = 3\,m$

$D = 1.5\,m$

$A = \frac{\pi}{4} \times D^2 = \frac{\pi}{4} \times 1.5^2$

$= 1.767\,m^2$

$I_{xx} = \frac{\pi D^4}{64} = \frac{\pi \times 1.5^4}{64} = 0.2485\,m^4$

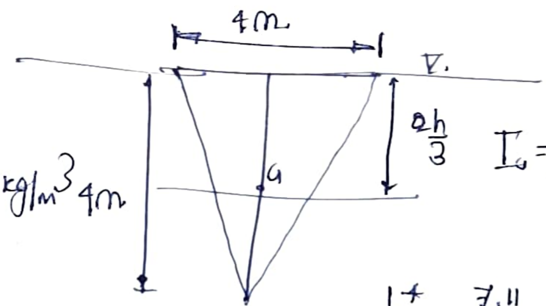
$F = \rho g \bar{h} A = 1000 \times 9.81 \times 3 \times 1.767 = 52\,kN.$

$\boxed{F = 52\,kN}$

$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{0.2485}{1.767 \times 3} + 3 = 3.0468\,m.$

$\boxed{h^* = 3.0468\,m}$

3. Determine the total pressure and center of pressure on an isoscales triangular plate of base 4 m and altitude 4 m when it is immersed vertically in an oil of specific gravity 0.9. The base of the plate is coincides with the free surface of oil.



$S = 0.9$

$\rho = S \times 1000$

$= 0.9 \times 1000 = 900\,kg/m^3$

$\bar{h} = \frac{h}{3} = \frac{4}{3}\,m.$

$A = \frac{1}{2} \times b \times h = \frac{1}{2} \times 4 \times 4 = 8\,m^2$

$F = \rho g \bar{h} A = 900 \times 9.81 \times \frac{4}{3} \times 8 = 94176\,N$

$= 94.176\,kN.$

$\frac{2h}{3}$

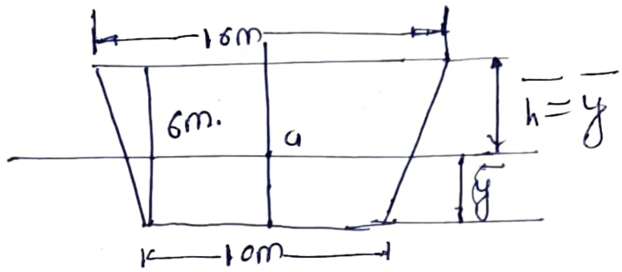
$I_G = \frac{bh^3}{36} = \frac{4 \times 4^3}{36} = 7.11\,m^4$

$h^* = \frac{7.11}{8 \times \frac{4}{3}} + \frac{4}{3}$

$= 2\,m.$

$\boxed{h^* = 2\,m.}$

4. A caisson for closing the entrance to a dry dock is of trapezoidal form 16m wide at the top and 10m wide at the bottom and 6m deep. Find the total pressure and center of pressure on ~~a car~~ the caisson if the water on the outside is just level with the top and the dock is empty.



$a = 10m$

$b = 16m.$

$h = 6\ m.$

$$A = \left(\frac{a+b}{2}\right) \times h$$

$$= \left(\frac{10+16}{2}\right) \times 6 = 13 \times 6$$

$$= 78m^2$$

$$\boxed{A = 78m^2}$$

$$\bar{I}_G = \left[\frac{a^2 + 4ab + b^2}{36(a+b)}\right] \times h^3 = \left[\frac{16^2 + 4 \times 16 \times 10 + 10^2}{36\,(16+10)}\right] \times 6^3$$

$$= 229.846m^4.$$

$$\boxed{\bar{I}_G = 229.846\ m^4.}$$

$$\bar{y} = \left(\frac{2a+b}{a+b}\right) \times \frac{h}{3} = \left(\frac{2 \times 16 + 16}{16+16}\right) \times \frac{6}{3}$$

$$= \frac{42}{32} \times 3 = \frac{42}{8}$$

$$\bar{y} = \left(\frac{2a+b}{a+b}\right) \times \frac{h}{3} = \left(\frac{2 \times 10 + 16}{10+16}\right) \times \frac{6}{3}$$

$$= \frac{36}{26} \times 2 = 2.769\ M.$$

$$\bar{h} = 6 - 2.769 = 3.231\ m.$$

$$\boxed{\bar{h} = 3.231\ M}$$

$$q = \frac{2}{3} C_d L \sqrt{2g}\, H^{3/2} + \frac{8}{15} C_d \tan(\theta/2) \sqrt{2g}\ \ldots$$

$$\bar{I}_G = \left[\frac{a^2 + 4ab + b^2}{36(a+b)}\right] \times h^3$$

$$= \left[\frac{10^2 + 4\times10\times16 + 16^2}{36\times(10+16)}\right] \times 6^3$$

$$= 229.846\, m^4.$$

$$\boxed{I_G = 229.846\, m^4.}$$

$$F = \rho g \bar{h} A = 1000 \times 9.81 \times \cancel{3.231}\, 2.769 \times 78$$

$$= 2.\cancel{472}\,118 \times 10^6\ N.$$

$$= 2.\cancel{472}\,118\ MN.$$

$$\boxed{F = 2.\cancel{472}\,118\ MN.}$$

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{229.864}{78 \times \cancel{3.231}\, 2.769} + \cancel{3.231}\, 2.769$$

$$= 3.833\, m$$

$$\boxed{h^* = 3.833\, m.}$$

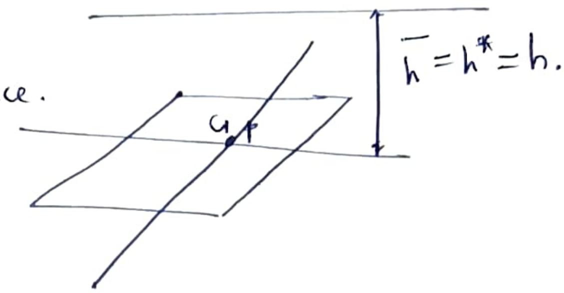
HORIZONTAL PLANE SURFACE SUB-MERGED IN LIQUID:

A horizontal plane surface immersed in a static liquid.

Every point of the surface is at the same depth from the free surface of the liquid the pressure intensity will be equal on the entire surface and equal to $p = \rho g h$.

h = depth of water surface.

A = Total Area of the surface.

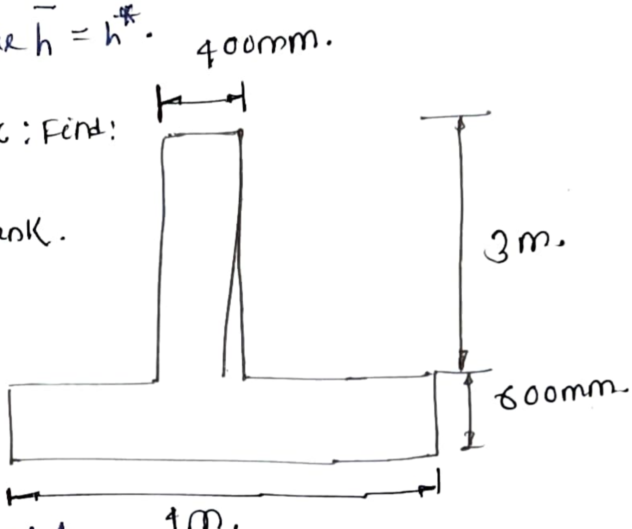


Total Force $F = p \times A$

$= \rho g h A = \rho g \bar{h} A$.

$$\boxed{F = \rho g \bar{h} A}$$

Here $\bar{h} = h^*$.



Ex:-1 Fig Shows a tank with full of water: Find:

(I). Total pressure on the bottom of the tank.

(II). Weight of water in the tank.

Width of the tank = 2m.

$A = \cancel{0.4 \times 3 + 0.6 \times 4}$ $4 \times 2 = 8m^2$.

$= \cancel{1.2 + 2.4 = 3.6m^2}$

$4 \times 2 \times 0.6 + 3 \times 2 \times 0.4$

Volume of the tank $V = \cancel{A \times L}$

$= \cancel{3.6 \times 2 = 7.2m^3}$

$= 7.2\ m^3$

$\bar{h} = 3.6m$.

Total pressure force: $F = \rho g \bar{h} A = 1000 \times 9.81 \times 8 \times 3.6$

$= 282528N$.

$= 282.528\ KN$.

$$\boxed{F = 282.528\ KN}$$

(II) Weight of water in the tank $W = wV$

$= \rho g V = 1000 \times 9.81 \times 7.2$

$= 70632N$.

KINEMATICS OF FLOW

(i) Kinematics is defined as that branch of science which deals with motion of particles without considering the forces causing the motion.

(ii). The velocity at any point in a flow field at any time is studied in this branch of fluid mechanics.

(iii) Once the velocity is known, then the pressure distribution and hence forces acting on the fluid can be determined.

METHODS OF DESCRIBING FLUID MOTION:

The fluid motion is described in two methods

1. Lagrangian Method
2. Eulerian Method.

In Lagrangian method a single fluid particle is followed during its motion and its velocity, accelaration, density etc are described.

In Eulerian method the velocity, accelaration, pressure, density etc are described at a point in a flow field.

The Eulerian method is commonly used in fluid mechanics.

TYPES OF FLUID FLOWS:

1. Steady and unsteady flow
2. Uniform and Non-uniform flow
3. Laminar and Turbulent flow
4. compressible and incompressible flow
5. Rotational and irrotational flow
6. one, two and three - dimensional flow.

$$Q = \frac{2}{3} C_d L \sqrt{2g}\, H^{3/2} + \frac{8}{15} C_d \tan(\theta/2) \sqrt{2g}\, H^{5/2}$$

...METER ... $\sqrt{2gh}$ $(A_1 ^ 2)$

1. STEDY & UNSTEADY FLOW:

Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density etc at a point do not change w.r.t time.

Mathematically, $\frac{\partial v}{\partial t} = 0$, $\frac{\partial \rho}{\partial t} = 0$, $\frac{dq}{dt} = 0$

Unsteady flow is that type of flow in which the velocity, pressure or density at a point changes w.r.t time.

Mathematically $\frac{dv}{dt} \neq 0$, $\frac{dq}{dt} \neq 0$, $\frac{\partial \rho}{\partial t} \neq 0$.

2. UNIFORM & NON-UNIFORM FLOW:

Uniform flow is defined as that type of flow in which the velocity at any given time does not change w.r.t space (Length of the direction of flow).

Mathematically, $\frac{dv}{ds} = 0$.

dv = velocity

ds = Length of flow in directions.

Non-uniform flow is that type of flow in which the velocity at any given time changes w.r.t space.

Mathematically, $\frac{dv}{ds} \neq 0$.

3. LAMINAR FLOW

Laminar flow is the type of flow in which the fluid particles moves along well-defined paths or streamlines and all the streamlines are straight and parallel.

The particles move in laminas or layers gliding smoothly over adjacent layer.

This type of flow is called stream line flow or viscous flow.

4. **TURBULENT FLOW**

Turbulent flow is that type of flow in which the fluid particles move in a zigzag way.

Due to movement of fluid particles in a zig-zag way the eddies formation takes place which are responsible for high energy loss.

For a pipe flow, the type of flow is determined by a non-dimensional number is called Reynold's Number.

$$Re = \frac{VD}{\nu}$$

$$= \frac{\rho VD}{\mu}$$

$$\boxed{Re = \frac{\rho VD}{\mu}}$$

D = diameter of pipe

V = Mean velocity of flow in pipe

ν = Kinematic viscocity of fluid.

$Re < 2000$ The flow is called laminar flow.

$Re > 4000$ The flow is called turbulent flow.

Reynold's number lies b/w 2000-4000 is called transitional flow.

5. **COMPRESSIBLE FLOW**

Compressible flow is that type of flow in which the density of the fluid changes from point to point or the density of the fluid is not constant for the fluid.

$$\rho \neq const.$$

INCOMPRESSIBLE FLOW:

Incompressible flow is that type of flow in which the density is constant for the fluid flow

$$\rho = const.$$

$$q = \frac{2}{3} C_d L \sqrt{2g}\, H^{3/2} + \frac{8}{15} C_d \tan(\theta/2) \sqrt{2g}\, H$$

(A1A2)

Liquids are generally incompressible while gases are compressible.

ROTATIONAL FLOW :

Rotational flow is that type of flow in which the fluid particles while flowing along streamlines also rotate about their own axis.

IRROTATIONAL FLOW

The fluid particles while flowing along streamlines do not rotate about their own axis then that type of flow is called irrotational flow.

RATE OF FLOW OR DISCHARGE (q)

Rate of flow or Discharge is defined as the quantity of fluid flowing per second through a section of pipe or channel.

For an incompressible fluid the rate of flow or discharge is expressed as the weight of fluid flowing across the section per second.

For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section.

For liquids: $\boxed{Q = AV}$

(m^3/sec or Litres/second)

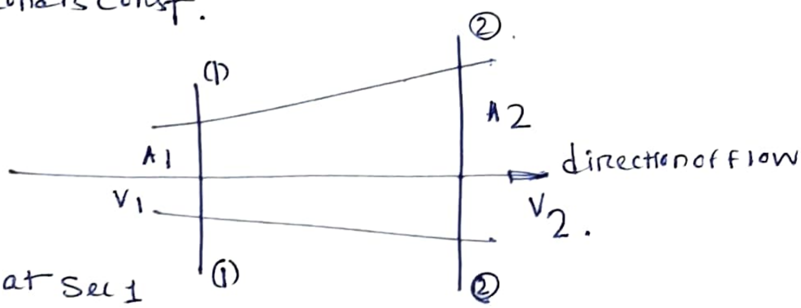
A = Cross-sectional Area of pipe

V = Average velocity of fluid across section

CONTINUITY EQUATION

The equation based on the principle of conservation of mass is called Continuity equation.

For a fluid flowing through the pipe at all the cross-section the quantity of fluid per second is const.



Rate of flow at Sec 1

$= \rho_1 A_1 V_1$

Rate of flow at sec 2 $= \rho_2 A_2 V_2$.

According to law of conservation of mass

Rate of flow at sec 1 = Rate of flow at sec. 2

$$\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$$

$$\boxed{A_1 V_1 = A_2 V_2}$$ For $\rho = \rho_1 = \rho_2$.

The equation is applicable to the compressible as well as incompressible fluids and is called continuity equation.

DYNAMICS OF FLUID FLOW

(I) The dynamics of fluid flow is the study of fluid motions with the forces causing flow.

(II) The dynamic behaviour of the fluid flow is analysed by the Newton's Second law of motion, which relates the accelaration with forces.

(III). The fluid is assumed to be incompressible and non-viscous.

According to Newton's Second law of motion

$$\boxed{F_N = ma.}$$

F_N = Net force acting on the fluid element
m = mass of the fluid element.
a = Accelaration.

In the fluid flow $\boxed{F_N = F_g + F_p + F_v + F_t + F_c.}$

F_g = gravity force
F_p = pressure Force
F_v = viscous Force
F_t = Force due to Turbulence
F_c = Force due to compressibility

Reynold's Equation of motion

$$\boxed{F_N = F_g + F_p + F_v + F_t}$$

(Fluid is incompressible). $F_c = 0$

Navier-stokes Equation of motion

$$\boxed{F_N = F_g + F_p + F_v}$$

$F_t = 0$
(Flow is laminar)

Euler's Equation of motion:

$$\boxed{F_N = F_g + F_p}$$

$F_v = 0.$
(Fluid having no viscosity) ie. Fluid is Ideal.

BERNOULLI'S EQUATION OF MOTION:

$$\boxed{E = \frac{P}{\rho g} + \frac{v^2}{2g} + Z}$$

$\frac{P}{\rho g}$ = pressure energy per unit weight of fluid or pressure head.
$\frac{v^2}{2g}$ = Kienietic energy per unit weight or kienietic head
Z = Potential head or Potential energy per unit weight.

Assumptions :-

(I) The fluid is ideal i.e. viscosity is Zero. ($\tau = 0$).

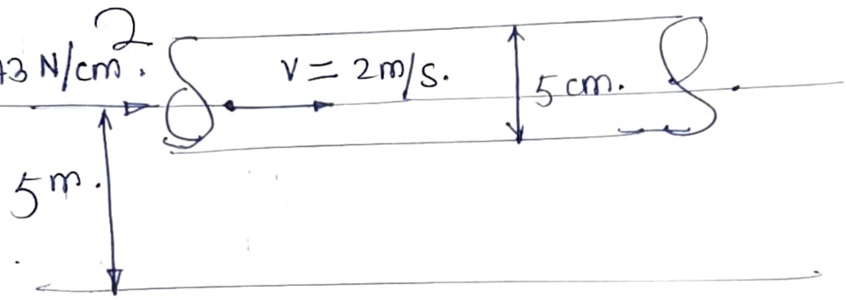
(II) The flow is steady

(III) The flow is incompressible (ρ = const.)

(IV) The flow is irrotational.

(1) Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm^2 with mean velocity 2 m/s. Find out the total head or total energy per unit weight of water at a cross-section which is 5 m above the datum line.

given,



$P = 29.43 \times 10^4 \, N/m^2$

$V = 2 \, m/s$

$d = 5 \, cm = 0.05 \, m.$

$Z = 5 \, m$ datum energy or datum head.

Pressure Head of water : $\dfrac{P}{\rho g} = \dfrac{29.43 \times 10^4}{1000 \times 9.81}$

$= 30 \, m.$

Velocity Head : $\dfrac{V^2}{2g} = \dfrac{2^2}{2 \times 9.81} = 0.204 \, m.$

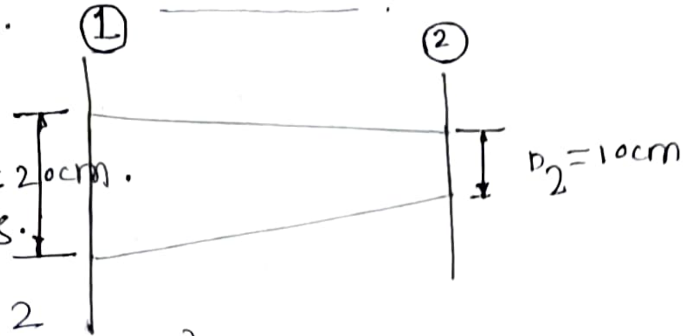
Total Head or Total Energy $E = \dfrac{P}{\rho g} + \dfrac{V^2}{2g} + Z$

$= 30 + 0.204 + 5$

$= 35.204 \, m.$

$$\boxed{E = 35.204 \, m}$$

2. A pipe through which water is flowing, is having diameters, 20cm and 10cm at the c/s 1 and 2 respectively. The velocity of water at Section 1 is given as 4 m/s. Find the velocity head at Sections 1 & 2 and also rate of discharge.



$D_1 = 20cm = 0.2m$

$D_2 = 10cm = 0.1m$ $V_1 = 4.0m/s$.

Velocity Head at sec. 1 $= \frac{V_1^2}{2g} = \frac{4^2}{2\times 9.81} = 0.815m.$

$A_1 = \frac{\pi}{4} \times D_1^2 = \frac{\pi}{4} \times 0.2^2 = 0.0314 m^2$

$A_2 = \frac{\pi}{4} \times D_2^2 = \frac{\pi}{4} \times 0.1^2 = 7.85 \times 10^{-3} m^2$

Discharge

$Q = AV$ As per continuity Eq^n

$A_1 V_1 = A_2 V_2$.

$Q = A_1 V_1$

$= 4 \times 0.0314 = 0.1256 m^3/sec.$

$V_2 = \frac{A_1 V_1}{A_2} = \frac{0.0314}{7.85 \times 10^{-3}} \times 4 = 16 m/s.$

Velocity head at sec. 2 $= \frac{V_2^2}{2g} = \frac{16^2}{2\times 9.81} = 13.047m.$

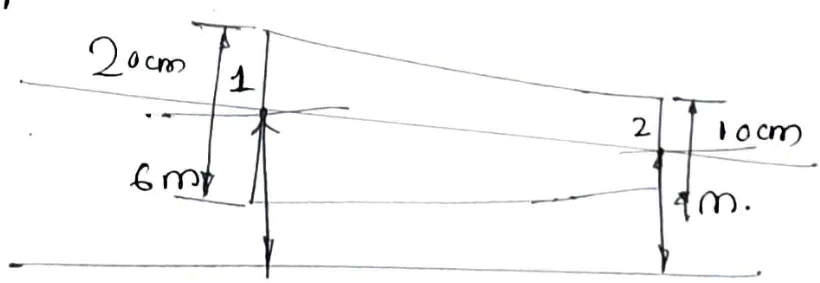
3. The water is flowing through a pipe having diameters 20cm and 10cm at sec. 1 and 2 respectively. The rate of flow through pipe is 35 Ltr/sec. The sec. 1 is 6m above datum line and section 2 is 4m above datum. If the pressure at sec. 1 is $39.24 N/cm^2$, Find the intensity of pressure at sec. 2.

$D_1 = 20cm = 0.2m$

$D_2 = 10cm = 0.1m.$

$Z_1 = 6m$

$Z_2 = 4m$



(4)

$Q = 0.035\ m^3/sec.$ $\qquad A_1 = \frac{\pi}{4} \times 0.2^2 = 0.0314\ m^2$

$P_1 = 39.24 \times 10^4\ N/m^2$ $\qquad A_2 = \frac{\pi}{4} \times 0.1^2 = 7.85 \times 10^{-3}\ m^2$

$V_1 = \frac{Q}{A_1} = \frac{0.035}{0.0314} = 1.114\ m/s$

As per Bernoulli's Theorem at sec 1&2:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$V_2 = \frac{Q}{A_2} = \frac{0.035}{7.85 \times 10^{-3}} = 4.45\ m/s.$

$$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{1.114^2}{2 \times 9.81} + 6 = \frac{P_2}{1000 \times 9.81} + \frac{4.45^2}{2 \times 9.81} + 4$$

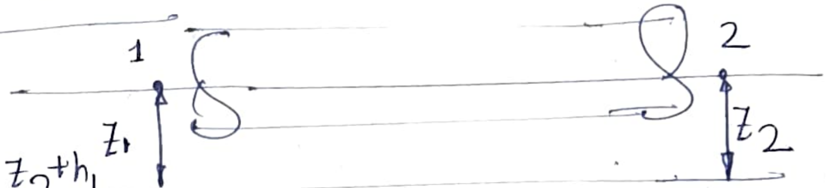
$$\left[\frac{39.24 \times 10^4}{10^3 \times 9.81} + \left(\frac{1.114^2 - 4.45^2}{2 \times 9.81}\right) + (6-4)\right] \times 9.81 \times 10^3 = P_2$$

$P_2 = 40.273 \times 10^4\ N/m^2$

$$\boxed{P_2 = 40.27\ N/cm^2}$$

The Bernoulli's Equation was derived on the assumption that the fluid is inviscid and therefore frictionless. (Non-viscous)

Bernoulli's Equation for real fluid :-



$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$h_L = \left(\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1\right) - \left(\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2\right)$$

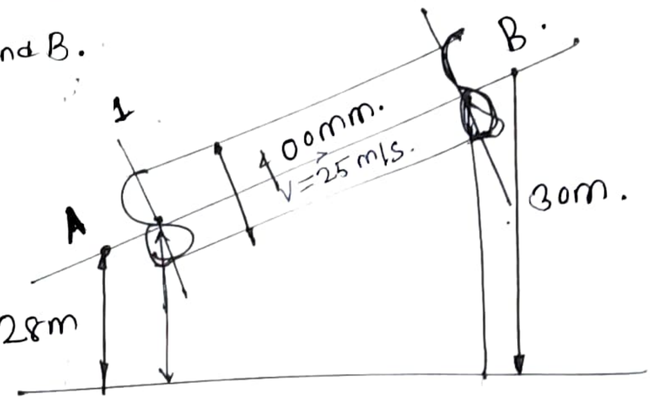
$= E_1 - E_2$ $\qquad h_L =$ Loss of energy b/w points 1&2

$$\boxed{h_L = E_1 - E_2}$$

(i) All the real fluids are viscous and hence offers resistance to flow.

(ii). There are some losses in fluid flows and hence in the application of Bernoulli's equation the losses have to be taken into consideration.

4. A pipe of diameter 400mm carries water at a velocity of 25 m/s. The pressure at the points A & B are given as 29.43 N/cm^2 and 22.563 N/cm^2 respectively while datum head at A and B are 28m and 30m. Find the loss of head b/w A and B.



$D = 400mm$
$= 0.4 m.$
$P_1 = 29.43 \times 10^4 \ N/m^2$
$P_2 = 22.563 \times 10^4 \ N/m^2$
$Z_1 = 28m$
$Z_2 = 30 m.$
$V = 25 m/s.$

$$E_1 = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 28 = 89.855 m.$$

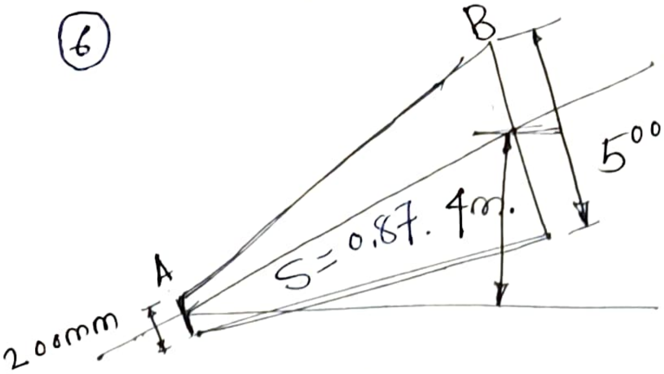
$$E_2 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 = \frac{29.563 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 30 = 84.855 m.$$

Loss of energy b/w point 1 & 2 = $E_1 - E_2$.
h_L $= 89.855 - 84.855$

$= 5 m.$

$$\boxed{h_L = 5m}$$

5. A pipeline carrying oil of specific gravity 0.87, changes in diameter from 200mm diameter at a position A to 500mm diameter at a position B which is 4m at a higher level. If the pressures at A and B are 9.81 N/cm^2 and 5.886 N/cm^2 respectively and the discharge is 200 Ltr/sec. Determine the loss of head and direction of flow.



$Q = 200\ \text{Litre/sec}$
$= 0.2\ m^3/\text{sec}.$
$D_1 = 200\,mm = 0.2\,m$
$D_2 = 500\,mm = 0.5\,m$

$P_1 = 9.81 \times 10^4\ N/m^2$
$P_2 = 5.886 \times 10^4\ N/m^2$

$S = 0.87$
$\rho = 0.87 \times 1000$
$= 870\ kg/m^3.$
$Z_1 = 0$
$Z_2 = 4\,m.$

$A_1 = \frac{\pi}{4} \times D_1^2 = \frac{\pi}{4} \times 0.2^2 = 0.0314\,m^2$
$A_2 = \frac{\pi}{4} \times D_2^2 = \frac{\pi}{4} \times 0.5^2 = 0.196\,m^2.$

$V_1 = \frac{Q}{A_1} = \frac{0.2}{0.0314} = 6.369\ m/s$
$V_2 = \frac{Q}{A_2} = \frac{0.2}{0.196} = 1.02\ m/s.$

at A

$$E_A = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{9.81 \times 10^4}{870 \times 9.81} + \frac{6.369^2}{2 \times 9.81} + 0 = 13.561\,m.$$

at B

$$E_B = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 = \frac{5.886 \times 10^4}{870 \times 9.81} + \frac{1.02^2}{2 \times 9.81} + 4 = 10.95\,m.$$

~~Loss of~~ Loss of head b/w point A & B

$h_L = E_A - E_B$
$= 13.561 - 10.95 = 2.611\,m.$

$$\boxed{h_L = 2.611\,m.}$$

BERNOULLI'S THEOREM :-

It states that in a steady, ideal flow of an incompressible fluid, the total energy at any point of fluid is constant.

The total energy consists of pressure energy, kinetic energy, potential energy or datum energy. (The energies per unit weight of the fluid)

$$\boxed{E = \frac{P}{\rho g} + \frac{V^2}{2g} + Z}$$

(7)

APPLICATIONS OF BERNOULLI'S EQUATION:

Bernoulli's equation is applied in all problems of incompressible fluid flow where energy considerations are involved.

Applications to the following measuring devices:

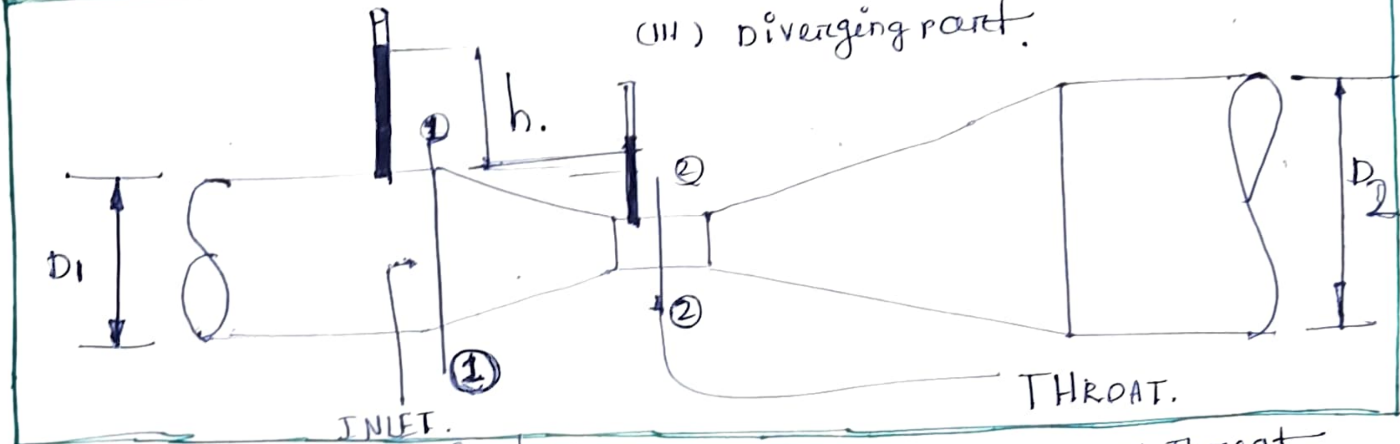
1. venturimeter
2. orificemeter
3. pitot tube.

1. VENTURIMETER

→ A venturimeter is fitted in a horizontal pipe through which a fluid is flowing (Water).

A venturimeter is a device used for measuring the rate of flow where energy considerations are involved.

It consists of three parts (I) A short converging part
(II) Throat
(III) Diverging part.



D_1 = diameter at inlet
P_1 = pressure at sec. 1
$A_1 = \frac{\pi}{4} \times D_1^2$
V_1 = velocity of fluid at sec. 1

D_2 = diameter at Throat
P_2 = pr. at sec. 2
$A_2 = \frac{\pi}{4} \times D_2^2$
V_2 = velocity of fluid at sec 2

Apply Bernoulli's equation at sec. 1 and 2:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$(\because Z_1 = Z_2$ pipe is horizontal.)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g}.$$

8

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}.$$

Apply continuity equation

$A_1 V_1 = A_2 V_2$

$$\boxed{V_1 = \frac{A_2 V_2}{A_1}}$$

$$= \frac{V_2^2}{2g} - \frac{(A_2 V_2/A_1)^2}{2g}.$$

$$= \frac{V_2^2}{2g}\left(1 - \frac{A_2^2}{A_1^2}\right).$$

$$h = \frac{P_1 - P_2}{\rho g}$$

= difference of pressure heads at section 1 2 2.

$$= \frac{V_2^2}{2g}\left(\frac{A_1^2 - A_2^2}{A_1^2}\right).$$

$$V_2^2 = 2gh\left(\frac{A_1^2}{A_1^2 - A_2^2}\right).$$

$$\boxed{V_2 = \sqrt{2gh}\left(\frac{A_1}{\sqrt{A_1^2 - A_2^2}}\right).}$$

Discharge $Q = A_2 V_2$

$$= A_2 . \sqrt{2gh}\left(\frac{A_1}{\sqrt{A_1^2 - A_2^2}}\right).$$

$$= \sqrt{2gh}\left(\frac{A_1 . A_2}{\sqrt{A_1^2 - A_2^2}}\right)$$

$$\boxed{Q_{Th.} = \sqrt{2gh}\left(\frac{A_1 . A_2}{\sqrt{A_1^2 - A_2^2}}\right)}.$$

It gives the discharge under ideal conditions and is called, Theoritical discharge.

C_d = coefficient of discharge / co-efficient venturimeter

$$C_d = \frac{Q_{Act.}}{Q_{Th.}}$$

Actual discharge $\Rightarrow$ $\boxed{Q_{Act.} = C_d . Q_{Th.} = C_d \times \left[\sqrt{2gh}\left(\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\right)\right]}$

CASE-I The differential manometer contains a liquid which is heavier than the liquid flowing through the pipe.

S_h = sp. gravity of the heavier liquid

S_o = sp. gravity of liquid flowing through pipe.

x = difference of heavier liquid column in U-Tube.

$$\boxed{h = x\left(\frac{S_h}{S_o} - 1\right)}$$

CASE-II The differential manometer contains a liquid which is lighter than the liquid flowing through the pipe

$$\boxed{h = x\left(1 - \frac{S_L}{S_o}\right)}$$

S_L = sp. gravity of the lighter liquid.

S_o = sp. gravity of the fluid flowing through the pipe

x = difference of the lighter liquid column in U-Tube.

U-Tube

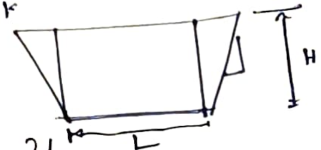
Inclined venturimeter with differential manometer:

CASE-III
$$h = \left(\frac{P_1}{\rho g} + Z_1\right) - \left(\frac{P_2}{\rho g} + Z_2\right) = x\left(\frac{S_h}{S_o} - 1\right)$$

differential manometer contains heavier liquid

CASE-IV
$$h = \left(\frac{P_1}{\rho g} + Z_1\right) - \left(\frac{P_2}{\rho g} + Z_2\right) = x\left(1 - \frac{S_L}{S_o}\right)$$

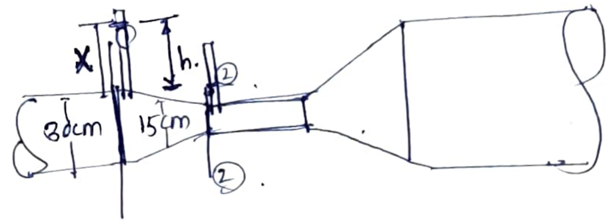
differential manometer contains a liquid which is lighter the liquid flowing through the pipe.



$$q = \frac{2}{3} C_d L\sqrt{2g}\, H^{3/2} + \frac{8}{15} C_d \tan(\theta/2)\sqrt{2g}\, H^{5/2}.$$

(10)

6. A horizontal venturimeter with inlet and throat diameters 30cm and 15cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and throat is 20 cm of mercury. Determine the rate of flow. $C_d = 0.98$



$h = 20$ cm of Hg.

$C_d = 0.98.$

$D_1 = 30\text{ cm} = 0.3\text{ m}$

$D_2 = 15\text{ cm} = 0.15\text{ m}$

$x = 20\text{ cm} = 0.2$

$S_h = 13.6$

$S_o = 1$

$$h = 0.2 \times \left(\frac{13.6}{1} - 1\right)$$

$= 2.52$ m of water

$$\boxed{h = 2.52\text{ m}}$$

$$A_1 = \frac{\pi}{4} \times D_1^2 = \frac{\pi}{4} \times 0.3^2 = 0.07\text{ m}^2$$

$$A_2 = \frac{\pi}{4} \times D_2^2 = \frac{\pi}{4} \times 0.15^2 = 0.017\text{ m}^2$$

Actual discharge: $Q_{act.} = C_d\sqrt{2gh}\left(\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\right).$

$$= 0.98 \times \sqrt{2 \times 9.81 \times 2.52} \times \left(\frac{0.07 \times 0.017}{\sqrt{0.07^2 - 0.017^2}}\right)$$

$= 0.12\text{ m}^3/\text{sec}$

$$\boxed{Q_{Act} = 0.120\text{ m}^3/\text{sec.}}$$

$$Q_{Th.} = \sqrt{2gh}\left(\frac{A_1 \cdot A_2}{\sqrt{A_1^2 - A_2^2}}\right)$$

$= 0.1232\text{ m}^3/\text{sec}$

Theoritical discharge.

$$\boxed{Q_{Th} = 0.1232\text{ m}^3/\text{sec.}}$$

7. An oil of sp. gravity 0.8 is flowing through a venturimeter having inlet diameter 20 cm and throat diameter 10 cm. The oil mercury differential manometer shows a reading of 25 cm. Calculate the discharge of oil through the horizontal venturimeter. $C_d = 0.98$.

$D_1 = 20\,cm = 0.2\,m \qquad A_1 = \frac{\pi}{4} \times 0.2^2 = 0.0314\,m^2$

$D_2 = 10\,cm = 0.1\,m \qquad A_2 = \frac{\pi}{4} \times 0.1^2 = 7.853 \times 10^{-3}\,m^2$

$x = 25\,cm.$

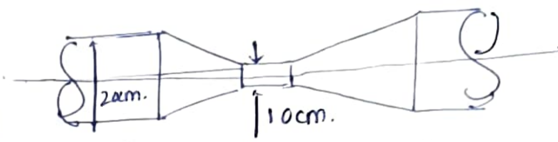
$S_h = 13.6 \qquad h = x\left(\frac{S_h}{S_o} - 1\right)$

$S_o = 0.8 \quad \rho = 800\,kg/m^3$

$= 0.25\left(\frac{13.6}{0.8} - 1\right)$

$C_d = 0.98.$

$= 4\,m$ of water.



S_h = specific gravity of heavy liquid

S_o = specific gravity of light liquid.

$$Q_{Th.} = \sqrt{2gh}\left(\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\right).$$

$$= \sqrt{2 \times 9.81 \times 4}\left(\frac{0.0314 \times 7.853 \times 10^{-3}}{\sqrt{0.0314^2 - (7.853 \times 10^{-3})^2}}\right).$$

$$= 71.852 \times 10^{-3}\,m^3/sec.$$

$$C_d = \frac{Q_{Act}}{Q_{Th.}}$$

$$Q_{Act} = C_d\,Q_{Th.}$$

$$= 0.98 \times 71.852 \times 10^{-3} = 70.415 \times 10^{-3}\,m^3/sec.$$

$$\boxed{Q_{Act.} = 70.415 \times 10^{-3}\,m^3/sec.}$$

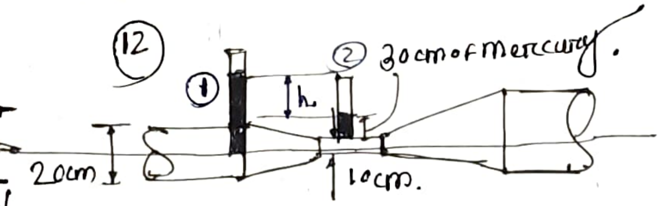
8. A horizontal venturimeter is with inlet diameter 20 cm and throat diameter 10 cm is used to measure the flow of water. The pressure at inlet is 17.658 N/cm² and the vaccum pressure at the throat is 30 cm of mercury. Find the discharge of water through venturimeter. $C_d = 0.98$.

$$q = \frac{2}{3} C_d\, L\sqrt{2g}\, H^{3/2} + \frac{8}{15} C_d \tan(\theta/2)\sqrt{2g}\, H^{5/2}$$

(12)

$D_1 = 20\,cm = 0.2\,m$

$D_2 = 10\,cm = 0.1\,m$



$17.658\,N/cm^2$

$p_1 = 17.658\,N/cm^2$

$= 17.658 \times 10^4\,N/m^2$

$A_1 = \frac{\pi}{4} \times 0.2^2 = 0.0314\,m^2$

$A_2 = \frac{\pi}{4} \times 0.1^2 = 7.853 \times 10^{-3}\,m^2$.

$$h_1 = \frac{P_1}{\rho g} = \frac{17.658 \times 10^4}{1000 \times 9.81}$$

$= 18m.$

~~Vaccum pressure at throat~~

$\cancel{P_2 = 30\,cm\ of\ mercury.}$

$\cancel{= -0.3\,m\ of\ mercury.}$

$\cancel{-0.3 = \frac{P_2}{\rho g} = h_2.}$

$\cancel{h_2 = -0.3 \times 13.6 = -4.08\,m\ of\ water.}$

$0.3 \times 13.6 = h \times 1$

$h = \frac{0.3 \times 13.6}{1}$

$= 4.08\,M.$

Vaccum pressure at Throat = 30cm of mercurry.

$h_2 = -0.3\,m$ of Hg.

$= -4.08\,m$ of water.

OR $P_2 = 13600 \times 9.81 \times \left(\frac{30}{100}\right)$

$= -(40.0248 \times 10^3)\,N/m^2$

$h_2 = \frac{P_2}{\rho g}$

$= \frac{-(40.0248 \times 10^3)}{1000 \times 9.81}$

$= -4.08\,m.$

$\boxed{h_2 = -4.08m}$

differential pressure head; $h = h_1 - h_2$

$= 18 - (-4.08)$

$= 22.08\,m.$

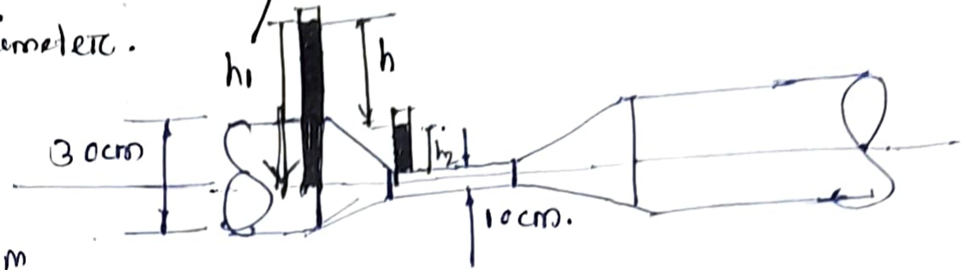
$\boxed{h = 22.08m}$

$$Q_{Act} = C_d \times \sqrt{2gh} \times \left(\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\right).$$

$$= 0.98 \times \sqrt{2 \times 9.81 \times 22.08} \times \left(\frac{0.0314 \times 7.853 \times 10^{-3}}{\sqrt{0.0314^2 - (7.853 \times 10^{-3})^2}}\right).$$

$= 0.165\,m^3/sec.$ $\boxed{Q_{Act} = 0.165\,m^3/sec}$

(9.) The inlet & throat diameter of a horizontal venturimeter are 30 cm and 10 cm respectively. The liquid flowing through the meter is water. The pressure intensity at inlet is 13.734 N/cm^2 while the vaccum pressure head at the throat is 37 cm of mercury. Find the rate of flow. Assume that 4% of differential head is lost b/w inlet and throat. Find also the value of Cd for venturimeter.



$D_1 = 30cm = 0.3m$

$D_2 = 10cm = 0.1m.$

$P_1 = 13.734 \times 10^4 \; N/m^2.$

$A_1 = \frac{\pi}{4} \times 0.3^2 = 0.0706 m^2$

$A_2 = \frac{\pi}{4} \times 0.1^2 = 7.853 \times 10^{-3} m^2$

$$h_1 = \frac{13.734 \times 10^4}{1000 \times 9.81}$$

$= 14 m.$

$h_2 = 37$ cm of mercury

$= -0.37 \times 13.6$

$= -5.032$ m of water.

differential head: $h = h_1 - h_2$

$= 14 + 5.032$

$= 19.032 m.$

$\boxed{h = 19.032 m}$

$\boxed{C_d = 0.98}$

$$Q_{Act.} = C_d \sqrt{2gh}\left(\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\right)$$

$$= 0.98 \times \sqrt{2 \times 9.81 \times 19.032} \times \left(\frac{0.0706 \times 7.853 \times 10^{-3}}{\sqrt{0.0706^2 - (7.853 \times 10^{-3})^2}}\right)$$

$= 0.1496 \; m^2/sec$

$\boxed{Q_{Act.} = 0.1496 \; m^3/sec}$

4% of differential head is lost b/w inlet and throat.

$h_L = 4\%$ of $h = 0.04 \times 19.032$

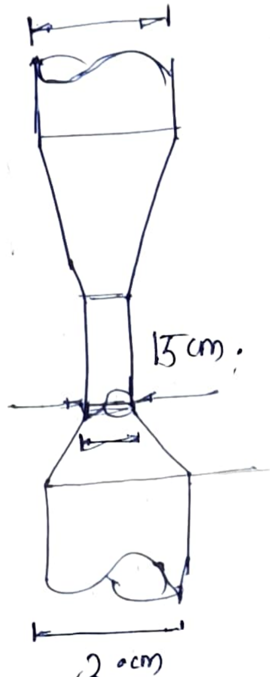
$= 0.7612 M.$

$$\boxed{C_d = \sqrt{\frac{h - h_L}{h}}} = \sqrt{\frac{19.032 - 0.7612}{19.032}} = 0.979 \simeq 0.98$$

$$q = \frac{2}{3} C_d L\sqrt{2g}\, H^{3/2} + \frac{8}{15} C_d \tan(\theta/2)\sqrt{2g}\, H^{5/2}.$$

(14)

(10) ~~A 30×15 cm~~ A 30 cm × 15 cm venturimeter is inserted in a vertical pipe carrying water, flowing in the upward direction. A differential mercury manometer connected to the inlet and throat gives a reading of 20 cm. Find the discharge. Take $C_d = 0.98$.



$D_1 = 30\,cm = 0.3\,m$ $\qquad A_1 = \frac{\pi}{4} \times 0.3^2 = 0.0706\ m^2$

$D_2 = 15\,cm = 0.15\,m.$ $\qquad A_2 = \frac{\pi}{4} \times 0.15^2 = 0.0176\ m^2.$

$C_d = 0.98$

$x = 20\,cm = 0.2\,m.$

$$h = x\left(\frac{S_h}{S_0} - 1\right)$$

$$= 0.2 \times \left(\frac{13.6}{1} - 1\right)$$

$$= 2.52\,m.$$

$$Q_{act} = C_d\sqrt{2gh}\left(\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\right)$$

$$= 0.98 \times \sqrt{2 \times 9.81 \times 2.52} \times \left(\frac{0.0706 \times 0.0176}{\sqrt{0.0706^2 - 0.0176^2}}\right)$$

$$= 0.1252\ m^3/sec$$

$$\boxed{Q_{act} = 0.1252\ m^3/sec}$$

$$q = \frac{2}{3} C_d L\sqrt{2g}\, H^{3/2} + \frac{8}{15} C_d \tan(\theta/2)\sqrt{2g}\, H$$

(18)

$$\frac{P_1 - P_2}{\rho g} + (z_2 - z_1).0 + \frac{V_2^2 - V_1^2}{2g} = h_L$$

$$\frac{V_2^2 - V_1^2}{2g} + 0.12 = \frac{0.2 \times V_1^2}{2g}$$

$$\boxed{\cancel{V_2^2 - V_1^2 = 0.12 \times 2 \times 9.81 = 2.35\,m}}$$

$$0.12 + \frac{V_1^2 - V_2^2 - 0.2V_1^2}{2g} = 0.$$

$$\boxed{0.12 + \frac{0.8V_1^2 - V_2^2}{2g} = 0}$$

Apply continuity eq^n b/w (1) & (2).

$$A_1V_1 = A_2V_2$$

$$V_1 = \frac{A_2}{A_1} \times V_2 = \frac{1}{0.0706} \cdot \frac{}{0.0176} \times V_2$$

$$= 4V_2/4$$

$$\boxed{4V_1 = V_2} \qquad \boxed{0.25V_2 = V_1}$$

$$0.12 + \frac{0.8V_1^2 - (4V_1)^2}{2g} = 0.$$

$$\frac{V_1^2(0.8^2 - 4^2)}{2 \times 9.81} = -0.12.$$

$$-0.782\, V_1^2 = -0.12$$

$$V_1 = \sqrt{\frac{0.12}{0.782}} = 0.4\ m/s$$

$$V_1 = 0.4\ m/s$$

$$V_2 = 4 \times 0.4 = 1.6\ m/s$$

Rate of Flow

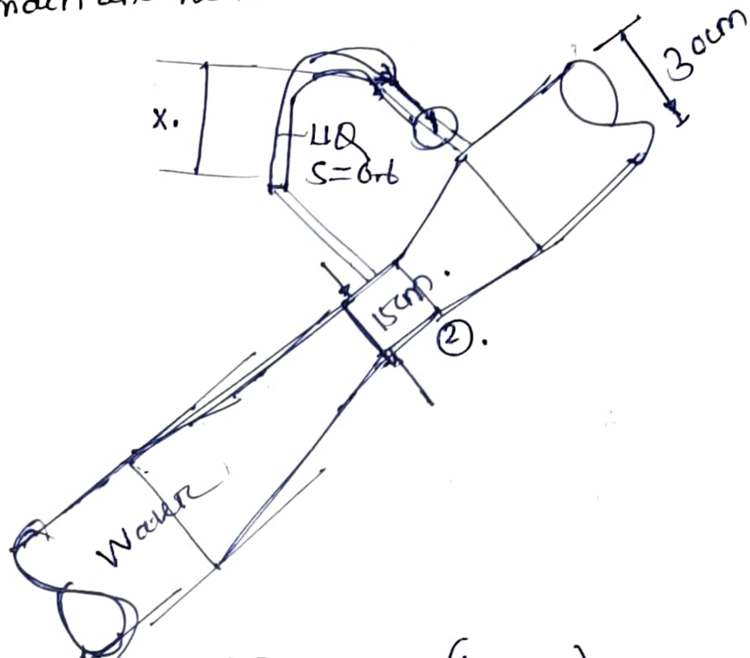
$$Q = A_1V_2 = A_2V_2$$

$$= 0.0176 \times 1.6$$

$$= 0.0281\ m^3/sec.$$

$$\boxed{Q = 0.0281\ m^3/sec}$$

(12). Find the discharge of water flowing through a pipe of 30 cm diameter placed in an inclined position where a venturimeter is inserted having a throat diameter 15 cm. The difference of pressure b/w main and throat is measured by a liquid of specific gravity 0.6 in an inverted U-tube which gives a reading of 30 cm. The loss of head b/w the main and the throat is 0.2 times the kinetic head of the pipe.



$P_1 = 30cm = 0.3m$

$P_2 = 0.15m.$

$A_1 = 0.0706\ m^2$

$A_2 = 0.0176\ m^2$

$S_L = 0.6 \quad S_0 = 1.$

$X = 30cm = 0.3m.$

difference of pressure head $\Rightarrow \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = h.$

$$h = x\left(1 - \frac{S_L}{S_0}\right).$$

$$= 0.3\left(1 - \frac{0.6}{1}\right)$$

$$= 0.3 \times 0.4 = 0.12m.$$

$$\boxed{h = 0.12m}$$

Loss of head $h_L = 0.2 \times$ kienietic head of pipe

$$= 0.2 \times \frac{v_1^2}{2g}. \qquad \boxed{h_L = \frac{0.2 v_1^2}{2g}}.$$

apply Bernoulli's Theorem b/w Sect 1 & 2.

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L$$

$$Q = \frac{2}{3} C_d L\sqrt{2g}\, H^{3/2} + \frac{8}{15} C_d \tan(\theta/2)\sqrt{2g}\, H^{5/2}$$

(16)

Apply continuity eq^n $A_1 V_A = A_2 V_B$.

$$V_A = \frac{A_2}{A_1} V_B$$

$$= \frac{5.026 \times 10^{-3}}{\cancel{7.853 \times 10^{-3}}\ 20.106 \times 10^{-3}} \times V_B = 0.64 V_B$$

$$\boxed{V_A = \cancel{0.64V_B}\ 0.25 V_B}$$

$$\frac{V_B^2 - V_A^2}{2g} = 0.75$$

$$V_B^2 - (0.25 V_B)^2 = 0.75 \times 2 \times 9.81$$

$$V_B = \sqrt{\frac{0.75 \times 2 \times 9.81}{1 - 0.25^2}}$$

$$= 4 \text{ m/s}$$

$$V_A = 0.25 \times 4 = 1 \text{ m/s}.$$

$$\boxed{\begin{aligned} V_B &= 4 \text{ m/s} \\ V_A &= 1 \text{ m/s}. \end{aligned}}$$

$$Q = A_1 V_A = A_2 V_B = 20.106 \times 1 \times 10^{-3} \text{ m}^3/\text{sec}$$

$$= 20.106 \times 10^{-3} \text{ m}^3/\text{sec}.$$

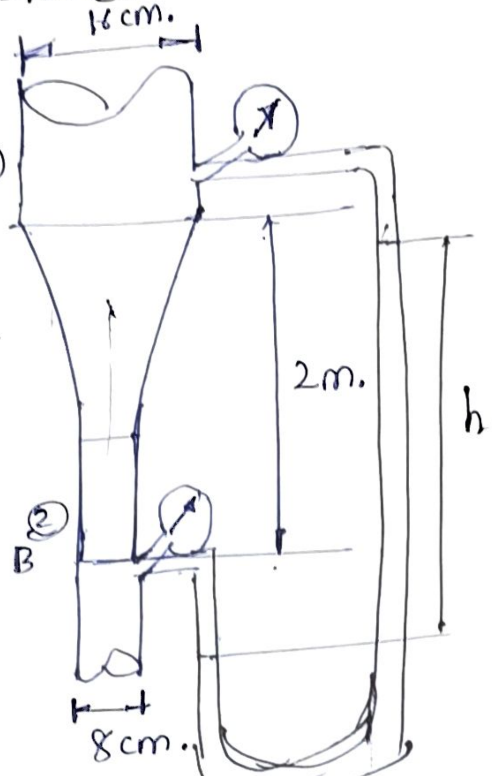
(ii) difference of level of mercury in the U-Tube:

$$\left(\frac{P_A - P_B}{\rho g}\right) + (Z_A - Z_B) = x\left(\frac{S_h}{S_o} - 1\right)$$

$$x = \frac{0.75}{\left(\frac{13.6}{0.8} - 1\right)} = 0.0468 \text{ m}$$

$$\boxed{x = 0.0468 \text{ m}}$$

(11) In a vertical pipe conveying oil of specific gravity 0.8. Two pressure gauges have been installed at A and B where the diameters are 16 cm and 8 cm respectively, A is 2 meters above B. The pressure gauge readings have shown that the pressure at B is greater than that A by $0.981\ N/cm^2$. ~~If the gauges at A and B are replaced by~~ Neglecting all losses, calculate the flow rate. If the gauges at A and B are replaced by tubes filled with the same liquid and connected to a U-Tube containing mercury. Calculate the difference of level of mercury in the two limbs of the U-Tube.



$D_1 = 16\ cm. = 0.16\ m$

$D_2 = 8\ cm. = 0.08\ m$

$P_B - P_A = 0.981\ N/cm^2 = 0.981 \times 10^4\ N/m^2$

$S = 0.8$

$\rho = 800\ kg/m^3$

$A_1 = \frac{\pi}{4} \times 0.16^2 = 20.106 \times 10^{-3}\ m^2$

$A_2 = \frac{\pi}{4} \times 0.08^2 = 5.026 \times 10^{-3}\ m^2$

Difference of pressure head $= \frac{P_B - P_A}{\rho g}$

$= \frac{0.981 \times 10^4}{800 \times 9.81} = 1.25\ m. \Rightarrow \boxed{\frac{P_B - P_A}{\rho g} = 1.25\ m}$

Apply Bernoulli's Theorem at point A & B.

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B$$

$$\frac{P_A - P_B}{\rho g} + (Z_A - Z_B) = \frac{V_B^2}{2g} - \frac{V_A^2}{2g}$$

$$-1.25 + (2 - 0) = \frac{V_B^2 - V_A^2}{2g}$$

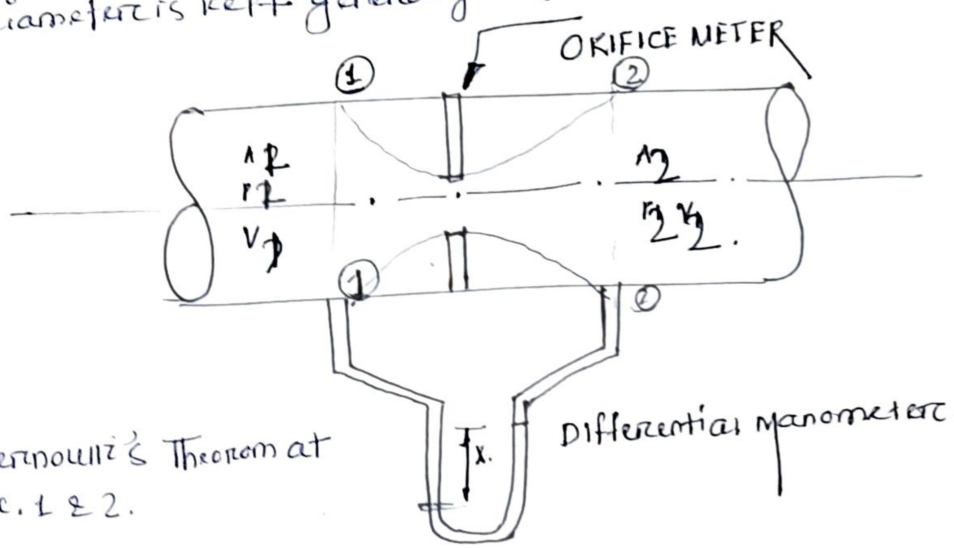
$$\boxed{\frac{V_B^2 - V_A^2}{2g} = 0.75\ m.}$$

ORIFICE METER OR ORIFICE PLATE

(I) It is a device used for measuring the rate of flow of a fluid through a pipe and it is a cheaper device as compared to venturimeter.

(II) It consists of a flat circular plate which has a circular sharp edged hole called orifice which is cocentric with the pipe.

(III). The orifice diameter is kept generally 0.5 times the diameter of the pipe.



Apply Bernoulli's Theorem at Sec. 1 & 2.

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2.$$

$$\left(\frac{P_1}{\rho g}+Z_1\right)-\left(\frac{P_2}{\rho g}+Z_2\right)=\frac{V_2^2-V_1^2}{2g}.$$

$$h=\frac{V_2^2-V_1^2}{2g}.$$

h = differential head.

$$V_2^2-V_1^2=2gh.$$

$$V_2=\sqrt{V_1^2+2gh}.$$

$$\boxed{V_2=\sqrt{V_1^2+2gh}.}$$

A_0 = Area at the orifice

A_c = Area at vena-contract

$$C_c=\frac{A_c}{A_0}\qquad \boxed{A_2=A_c=C_cA_0}$$

C_c = coefficient of contraction.

As per continuity eq^n

$$A_1V_1=A_2V_2.$$

$$Q = \frac{2}{3} C_d L\sqrt{2g}\, H^{3/2} + \frac{8}{15} C_d \tan(\theta/2)\sqrt{2g}\, H^{5/2}.$$

20

$$V_1 = \frac{A_2 V_2}{A_1} = \frac{C_c A_0 V_2}{A_1}.$$

$$\boxed{V_1 = \frac{C_c A_0 V_2}{A_1}}$$

$$V_2 = \sqrt{2gh + V_1^2}$$

$$= \sqrt{2gh + \left(\frac{C_c A_0 V_2}{A_1}\right)^2}$$

$$V_2^2 = 2gh + \left(\frac{A_0}{A_1}\right)^2 . C_c^2 . V_2^2$$

$$V_2^2 - C_c^2 \left(\frac{A_0}{A_1}\right)^2 V_2^2 = 2gh.$$

$$V_2^2 \left[1 - C_c^2\left(\frac{A_0}{A_1}\right)^2\right] = 2gh.$$

$$V_2 = \sqrt{\frac{2gh}{\left[1 - C_c^2\left(\frac{A_0}{A_1}\right)^2\right]}}.$$

$$\boxed{V_2 = \frac{\sqrt{2gh}}{\sqrt{1 - C_c^2\left(\frac{A_0}{A_1}\right)^2}}}$$

The discharge : $Q = A_2 V_2.$

$$= \frac{C_c A_0 \sqrt{2gh}}{\sqrt{1 - C_c^2\left(\frac{A_0}{A_1}\right)^2}} = \frac{C_c A_0\sqrt{2gh}}{\sqrt{1 - C_c^2\left(\frac{A_0}{A_1}\right)^2}}$$

$$\boxed{Q = \frac{C_c A_0\sqrt{2gh}}{\sqrt{1 - C_c^2\left(\frac{A_0}{A_1}\right)^2}}}$$

$$C_d = C_c \frac{\sqrt{1-\left(\frac{A_o}{A_1}\right)^2}}{\sqrt{1-\left(\frac{A_o}{A_1}\right)^2 C_c^2}}$$

$$\boxed{C_c = C_d \frac{\sqrt{1-\left(\frac{A_o}{A_1}\right)^2 C_c^2}}{\sqrt{1-\left(\frac{A_o}{A_1}\right)^2}}}$$

$$Q = A_2 V_2 = A_o C_c V_2$$

$$= C_d \frac{\sqrt{1-\left(\frac{A_o}{A_1}\right)^2 C_c^2}}{\sqrt{1-\left(\frac{A_o}{A_1}\right)^2}} \cdot A_o \cdot \frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{A_o}{A_1}\right)^2 C_c^2}}$$

$$= \frac{C_d \cdot A_o \sqrt{2gh}}{\sqrt{1-\left(\frac{A_o}{A_1}\right)^2}}$$

$$= \frac{C_d \cdot A_o A_1 \sqrt{2gh}}{\sqrt{A_1^2 - A_o^2}}$$

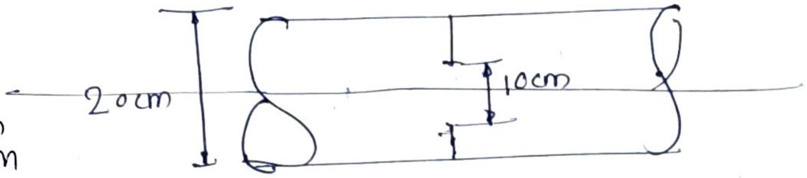
$$\boxed{Q = \frac{C_d A_o A_1 \sqrt{2gh}}{\sqrt{A_1^2 - A_o^2}}}$$

C_d = coefficient of discharge for orificemeter.

* Once you start liking someone, their mere ~~feeling~~ presence evokes a warm feeling within you.

$$q = \frac{2}{3} C_d L\sqrt{2g}\, H^{3/2} + \frac{8}{15} C_d \tan(\theta/2)\sqrt{2g}\, H^{5/2}.$$

An orificemeter with orifice diameter 10cm is inserted in a pipe of 20cm diameter. The pressure gauges fitted up stream and downstream of the orificemeter, gives reading of $19.62\,N/cm^2$ and $9.81\,N/cm^2$ respectively. Coefficient of discharge of orificemeter is given as 0.6. Find the discharge of water through the pipe.



$D_1 = 20cm$
$= 0.2M$

$D_0 = 0.1\,m.$

$A_1 = \frac{\pi}{4} \times 0.2^2 = 0.0314\,m^2$

$A_0 = \frac{\pi}{4} \times 0.1^2 = 7.853 \times 10^{-3}\,m^2.$

$P_1 = 19.62\,N/cm^2$
$= 19.62 \times 10^4\,N/m^2$

$P_2 = 9.81 \times 10^4\,N/m^2.$

$C_d = 0.6$

$h_1 = \frac{P_1}{\rho g} = \frac{19.62 \times 10^4}{1000 \times 9.81}$
$= 20m.$

$h_2 = \frac{P_2}{\rho g} = \frac{9.81 \times 10^4}{1000 \times 9.81}$
$= 10m.$

$h = h_1 - h_2$
$= 20 - 10 = 10M.$

$\boxed{h = 10m}$

$$q = \frac{C_d\sqrt{2gh}\, A_1 A_0}{\sqrt{A_1^2 - A_0^2}}$$

$$= \frac{0.6\sqrt{2 \times 9.81 \times 10} \times 0.0314 \times 7.853 \times 10^{-3}}{\sqrt{0.0314^2 - (7.853 \times 10^{-3})^2}}$$

$= 0.0681\,m^3/sec.$

$\boxed{q = 0.0681\,m^3/sec.}$

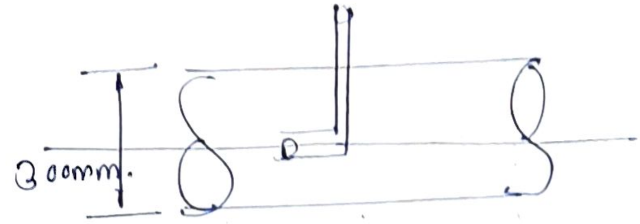
$\sqrt{2g}\, H^{3/2} + \frac{8}{\ } C_d \tan(\theta/2)\sqrt{2g}\ H$

A Pitot-static tube placed in the center of a 300mm pipeline has one orifice pointing upstream and other perpendicular to it. The mean velocity in the pipe is 0.80 of the central velocity. Find the discharge through the pipe if the pressure difference b/w the two orifices is 60mm of water. Take the coefficient of pitot tube $C_v = 0.98$.

$D = 300mm$
$= 0.3 m.$

$C_v = 0.98$

$h = 60mm$ of water
$= 0.06 m.$



$V_{act.} = C_v\sqrt{2gh} = 0.98 \times \sqrt{2\times 9.81\times 0.06}$

$= 1.063 m.$

$V_{mean} = 0.8 \times V_{act}$

$= 0.8 \times 1.063 = 0.85\ m/s.$

$\boxed{V = 0.85\ m/s.}$

Discharge : $Q = AV$

$= \left(\frac{\pi}{4}\times 0.3^2\right)\times 0.85 = 0.06\ m^3/sec.$

$\boxed{Q = 0.06\ m^3/sec.}$

Find the velocity of flow of an oil through a pipe, when the difference of mercury level in a differential U-Tube manometer connected to the two tappings of the pitot tube, is 100mm. Take co-efficient of pitot tube 0.98 and Sp. gravity of oil 0.8.

$x = 100mm = 0.1 m.$

$C_v = 0.98$

$S_g = 13.6$

$S_o = 0.8.$

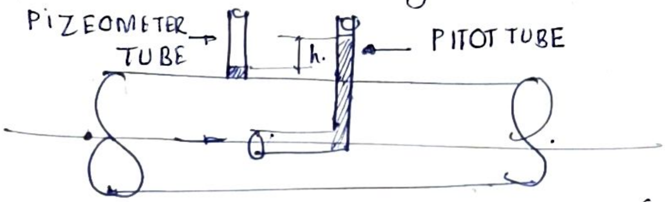
$h = x\left(\frac{S_g}{S_o} - 1\right)$

$= 0.1\times\left(\frac{13.6}{0.8} - 1\right) = 1.6 m.$

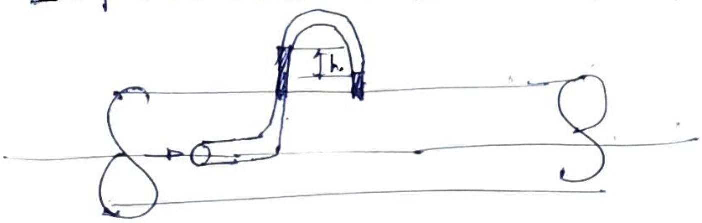
$V = C_v\sqrt{2gh} = 0.98\times\sqrt{2\times 9.81\times 1.6} = 5.5 m/s.$

Velocity of flow in a pipe by using pitot tube:

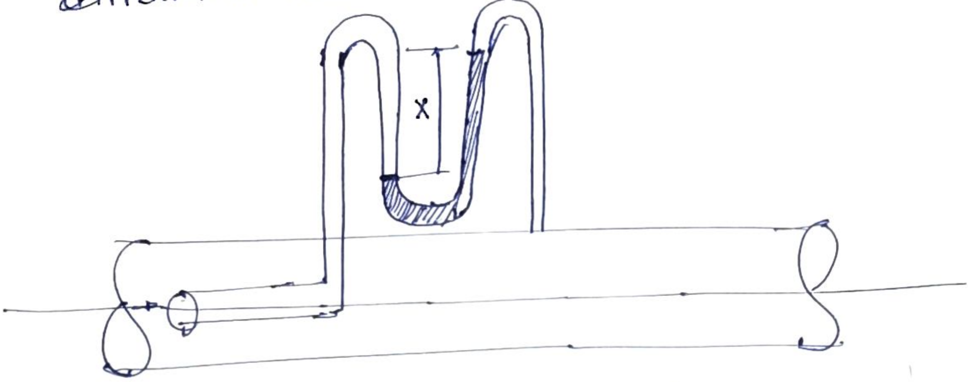
1. Pitot tube along with a vertical pizeometer tube



2. Pitot tube connected with pizeometer Tube:



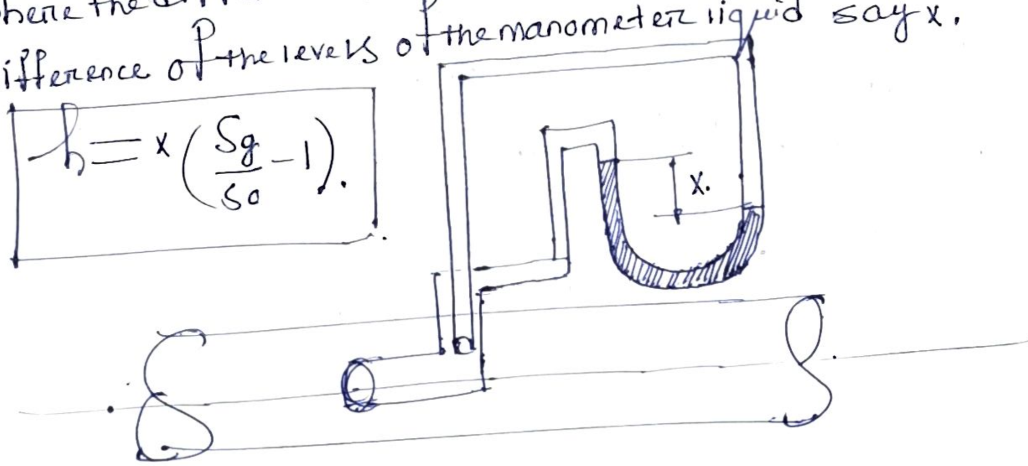
3. Pitot - Tube and Vertical pizeometer Tube connected with a U-Tube differential manometer ~~tube~~



4. Pitot static tube which consists of two circular cocentric tubes one inside the other with some annular space in between.

The outlet of the two tubes are connected to the differential manometer where the difference of pressure head 'h' is measured by knowing the difference of the levels of the manometer liquid say x.

$$\boxed{h = x\left(\frac{S_g}{S_o} - 1\right)}$$



$$= \frac{2}{3} C_d L \sqrt{2g}\, H^{3/2} + \frac{8}{15} C_d \tan(\theta/2) \sqrt{2g}\, H^{5/2}.$$

PITOT-TUBE

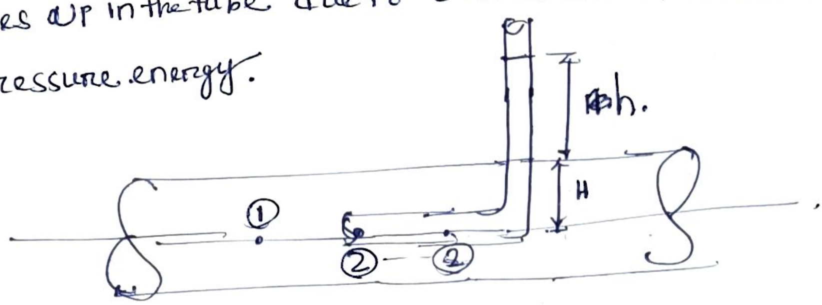
(I) It is a device used for measuring the velocity of flow at any point in a pipe or channel.

(II) It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of kinetic energy into pressure energy.

(III) The pitot tube consists of a glass tube bent at right angles.

(IV) The lower end which is bent through 90° is directed in the upstream direction.

(V) The liquid rises up in the tube due to conversion of kinetic energy into pressure energy.



Apply Bernoulli's Theorem at point 1 & 2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2.$$

$$H = \frac{P_1}{\rho g}$$

$$H + h = \frac{P_2}{\rho g}$$

$$Z_1 = Z_2.$$

$$V_2 = 0.$$

$$H + \frac{V_1^2}{2g} = H + h.$$

$$\frac{V_1^2}{2g} = h.$$

$$V_1 = \sqrt{2gh}$$

$$\boxed{V_1 = \sqrt{2gh}}$$ This is called theoritical velocity.

Actual velocity: $V = C_V . V_1$

$$= C_V \sqrt{2gh}$$

$$\boxed{V = C_V \sqrt{2gh}}$$

An orificemeter with orifice diameter 15 cm is inserted in a pipe of 30 cm diameter. The pressure difference measured by a mercury oil differential manometer on the two sides of the orificemeter gives a reading of 50 cm of mercury. Find the rate of flow of oil of sp. gravity 0.9 when coefficient of discharge of orificemeter is is 0.64.

$D_0 = 15\text{ cm} = 0.15\text{ m}$

$D_1 = 30\text{ cm} = 0.3\text{ m}$

$A_0 = 0.0176\text{ m}^2$

$A_1 = 0.0706\text{ m}^2$

$x = 50\text{ cm} = 0.5\text{ m}$

$S_h = 13.6$

$S_o = 0.9.$

$$h = x\left(\frac{S_h}{S_o} - 1\right)$$

$$= 0.5 \times \left(\frac{13.6}{0.9} - 1\right)$$

$$= 7.05\text{ m}. \quad \boxed{h = 7.05\text{ m}}$$

$C_d = 0.64.$

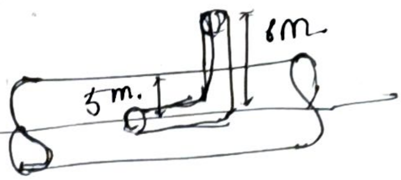
$$Q = \frac{C_d A_1 A_0 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$$

$$= \frac{0.64 \times 0.0706 \times 0.0176 \times \sqrt{2 \times 9.81 \times 7.05}}{\sqrt{0.0706^2 - 0.0176^2}}$$

$$= 0.137\text{ m}^3/\text{sec}.$$

$$\boxed{Q = 0.137\text{ m}^3/\text{sec}.}$$

A pitot static tube is used to measure the velocity of water in a pipe. The stagnation pressure head is 6m and static pressure head is 5m. Calculate the velocity of flow assuming the coefficient of tube equal to 0.98.



$h = 6-5 = 1m.$

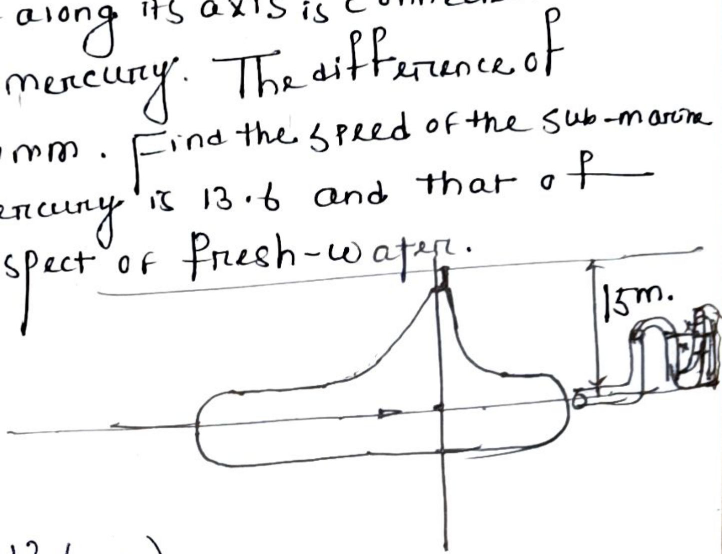
$C_v = 0.98.$

$V = C_v \times \sqrt{2gh} = 0.98\sqrt{2 \times 9.81 \times 1}$

$= 4.34 \text{ m/s}$

$\boxed{V = 4.34 \text{ m/s.}}$

A Sub-marine moves horizontally in sea and has its axis 15m below the surface of water. A pitot-tube properly placed just in front of the sub-marine and along its axis is connected to the two limbs of a U-Tube containing mercury. The difference of mercury level is found to be 170mm. Find the speed of the sub-marine knowing that the sp gravity of mercury is 13.6 and that of sea-water is 1.026 with respect of fresh-water.



$x = 170mm = 0.17 m.$

$S_g = 13.6$

$S_o = 1.026$

$h = x\left(\frac{S_g}{S_o} - 1\right) = 0.17\left(\frac{13.6}{1.026} - 1\right)$

$= 2.083 \text{ m.}$

$\boxed{h = 2.083 \text{ m}}$

$V = \cancel{C_v}\sqrt{2gh} = \sqrt{2 \times 9.81 \times 2.083}$

$= 6.4 \text{ m/s.}$

$\boxed{V = 6.4 \text{ m/s.}}$

$V = \frac{6.4 \times 3600}{1000}$

$= 23 \text{ KM/Hr.}$

$\boxed{V = 23 \text{ KM/Hr}}$

$H^{3/2} + 8\_\, C_d \tan(\theta/2)\sqrt{2g}\, H^{5/2}$.

A pitot-tube is inserted in a pipe of 300mm diameter. The static pressure in pipe is 100mm of mercury. The stagnation pressure at the center of the pipe, recorded by the pitot-tube is $0.981\ N/cm^2$. Calculate the rate of flow of water through pipe. If the velocity of flow is 0.85 times the central velocity. Take $C_v = 0.98$.

$D = 300mm = 0.3m$.

$C_v = 0.98$

$h_1 = 100mm$ of mercury $\quad P_1 = 100mm$ of mercury $= \left[13600 \times 9.81 \times \frac{100}{1000}\right]$

$= \cancel{13.6 \times 1000 \times}\ 0.1 \times 13.6 = -1.36m.$

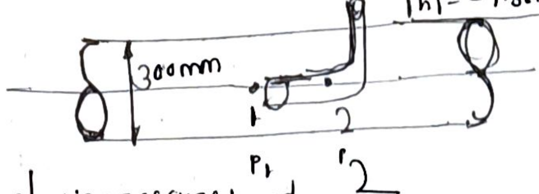
$= -13.3416 \times 10^3\ N/m^2$

$h_1 = \frac{-(13.3416 \times 10^3)}{1000 \times 9.81} = -1.36m$

$\boxed{h_1 = -1.36m}$

$P_2 = 0.981\ N/cm^2$

$= 0.981 \times 10^4\ N/m^2$



$h_2 = \frac{0.981 \times 10^4}{1000 \times 9.81} = 1m.$

$h =$ stagnation pressure head − static pressure head.

$h = h_2 - h_1 = \cancel{1.36 + 1}\ 1 - (-1.36).$

$= \cancel{2.36m}.\ 2.36$

$\boxed{h = \cancel{2.36m}.\ 2.36m.}$

$V = C_v \sqrt{2gh}$

$= 0.98\sqrt{2 \times 9.81 \times 2.36}$

$= 6.67\ m/s.$

$\bar{V} = 0.85V$

$= 0.85 \times 6.67$

$= 5.67\ m/s.$

$\boxed{\bar{V} = 5.67 m/s.}$

$Q = \left(\frac{\pi}{4} \times 0.3^2\right) \times 5.67 = 0.4\ m^3/sec$

$\boxed{Q = 0.4\ m^3/sec}$

Crude oil of specific gravity 0.85 flows upwards at a volume rate of flow of 60 Ltrs/sec through a vertical venturimeter with an inlet diameter of 200mm and a throat diameter of 100mm. The coefficient of discharge of the venturimeter is 0.98.

The vertical distance b/w the pressure tappings is 300mm.

(I) If two pressure gauges are connected at the tappings such that they are positioned at the levels of their corresponding tapping points, Determine the difference of readings in N/cm² of the two pressure gauges.

(II) If a mercury differential manometer is connected, in place of pressure gauges, to the tappings such that the connecting tube upto mercury are filled with oil, determine the difference in the level of mercury column.

$S_o = 0.85 \qquad \rho = 0.85 \times 1000$
$= 850 \, kg/m^3$

$Q = 60 \text{ Ltrs/sec.}$
$= 0.06 \, m^3/sec.$

$D_1 = 200 \, mm = 0.2 \, m$

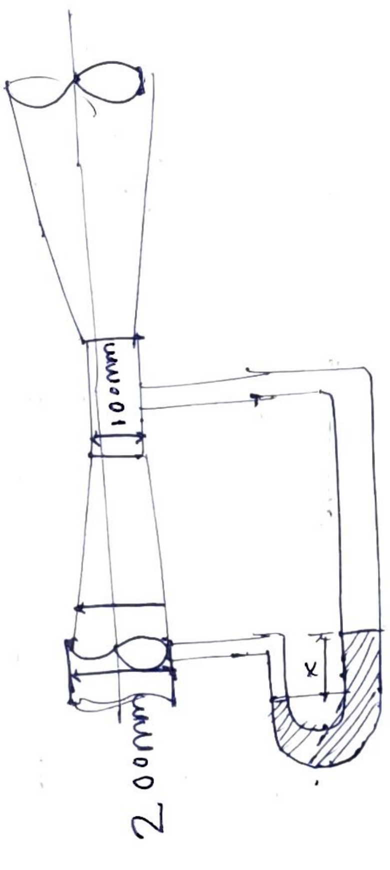
$D_2 = 100 \, mm = 0.1 \, m.$

$C_d = 0.98$

$A_1 = \frac{\pi}{4} \times 0.2^2 = 0.0314 \, m^2$

$A_2 = \frac{\pi}{4} \times 0.1^2 = 7.853 \times 10^{-3}$

$Z_2 - Z_1 = 300 \, mm = 0.3 \, m.$



$$Q = \frac{C_d \sqrt{2gh} \, A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$$

$$\ldots 3/2 + 8\, c\, \tan(\theta/2)\sqrt{2g}\, H^{5/2}.$$

$$0.06 = \frac{0.98 \times 0.0314 \times 7.853\times10^{-3}\sqrt{2\times9.81\times h}}{\sqrt{0.0314^2 - (7.853\times10^{-3})^2}}$$

$$h = \left[\frac{0.06 \times \left(\sqrt{0.0314^2 - (7.853\times10^{-3})^2}\right)}{0.98\times0.0314\times7.853\times10^{-4}\times\sqrt{2\times9.81}}\right]^2$$

$$h = 2.904\,m.$$

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right)$$

$$2.904 = \frac{p_1 - p_2}{\rho g} + (z_1 - z_2)$$

$$2.904 + 0.3 = \frac{p_1 - p_2}{\rho g}$$

$$p_1 - p_2 = (3.204) \times 850 \times 9.81$$

$$= 2.671\times10^4\ N/m^2$$

$$\boxed{\begin{aligned} p_1 - p_2 &= 2.671\times10^4\ N/m^2 \\ &= 2.671\ N/cm^2 \end{aligned}}$$

(ii)

$$h = x\left(\frac{S_H}{S_o} - 1\right)$$

$$2.904 = x\left(\frac{13.6}{0.85} - 1\right)$$

$$x = \frac{2.904}{15}$$

$$= 0.1936\ m.$$

$$\boxed{x = 19.36\ cm\ \text{of oil}}$$

$$\ldots 3/2 + \frac{8}{15} C_d \tan(\theta/2)\sqrt{2g}\, H^{5/2}.$$

$$P_2 = 5.443 \times 10^4 \, N/m^2$$

$$\boxed{P_2 = 5.443 \, N/cm^2}$$

$$F_x = P_1 A_1 - P_2 A_2 \cos\theta + \rho Q (V_1 - V_2 \cos\theta)$$

$$= 8.829\times10^4 \times 0.2827 - 5.443\times10^4 \times 0.0706 \times \cos(45^\circ) + 1000\times 0.6\,[2.122 - 8.498\cos(45^\circ)]$$

$$= 19.91\times10^3 \, N.$$

$$\boxed{F_x = 19.91\times10^3 \, N.}$$

$$F_y = -(\rho Q V_2 \sin\theta + P_2 A_2 \sin\theta)$$

$$= -[1000\times0.6\times8.498\times\sin(45^\circ) + 5.443\times10^4\times0.0706\times\sin(45^\circ)]$$

$$= -6.347\times10^3 \, N.$$

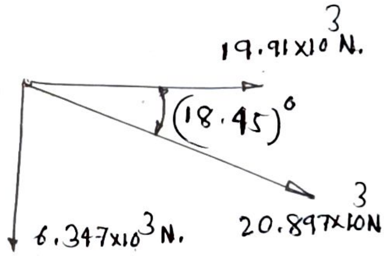
$$\boxed{F_y = -6.347\times10^3 \, N.}$$

Resultant force: $F_R = \sqrt{F_x^2 + F_y^2}$

$$= \sqrt{(19.91\times10^3)^2 + (-6.347\times10^3)^2}$$

$$= 20.897\times10^3 \, N.$$

$$\boxed{F_R = 20.897\times10^3 \, N.}$$



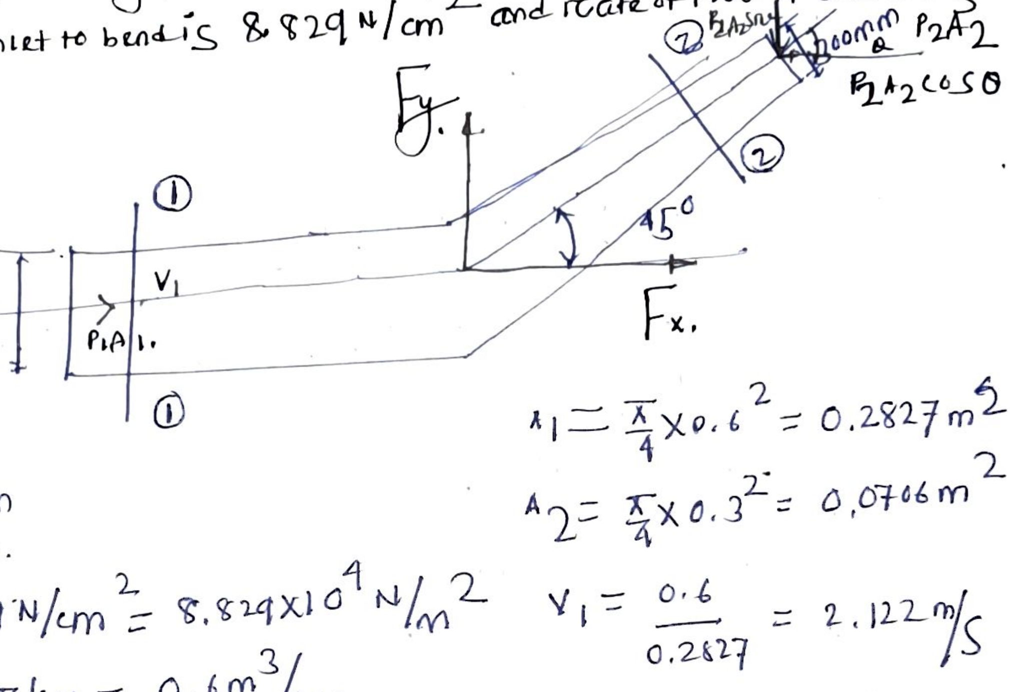
$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(-\frac{6.347\times10^3}{19.91\times10^3}\right) = -(18.45)^\circ$$

The Resultant Force $\boxed{F_R = \sqrt{F_x^2 + F_y^2}}$

Angle made by the resultant force with horizontal direction is given by.

$$\boxed{\tan\theta = \frac{F_y}{F_x} \qquad \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)}$$

1. A 45° reducing bend is connected in a pipeline, the diameters at the inlet and outlet of the bend being 600mm and 300mm respectively. Find the force exerted by the water on the bend if the intensity of pressure at inlet to bend is $8.829\ N/cm^2$ and rate of flow of water is 600 Ltrs/sec.



$\theta = 45^\circ$

$D_1 = 600mm = 0.6m$

$D_2 = 300mm = 0.3m$

$P_1 = 8.829\ N/cm^2 = 8.829\times10^4\ N/m^2$

$Q = 600\ Ltrs/sec = 0.6\ m^3/sec.$

$A_1 = \frac{\pi}{4}\times0.6^2 = 0.2827\ m^2$

$A_2 = \frac{\pi}{4}\times0.3^2 = 0.0706\ m^2$

$V_1 = \frac{0.6}{0.2827} = 2.122\ m/s$

$V_2 = \frac{0.6}{0.0706} = 8.498\ m/s.$

Apply Bernoulli's Equation at Sec. 1 & 2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$Z_1 = Z_2$

$$\left[\frac{8.829\times10^4}{1000\times9.81} + \frac{2.122^2 - 8.498^2}{2\times9.81}\right]\times1000\times9.81 = P_2$$

$$\ldots^{3/2} + \frac{8}{15} C_d \tan(\theta/2)\sqrt{2g}\; H^{5/2}.$$

(I) F_x and F_y be the components of the forces exerted by the fluid on the bend in x- and y- direction respectively.

(II). Then the force exerted by the bend on the fluid in the direction of x and y will be equal to F_x and F_y but in opposite directions.

(III). Component of force exerted by bend on the fluid in the x-direction $= -F_x$ and in the direction of $y = -F_y$.

(IV). The external forces acting on the fluid are P_1A_1 and P_2A_2 on the section 1 and 2 respectively.

The momentum equation in x-direction is given by:

$$\boxed{F_x = \text{Rate of change of momentum in x direction}}$$

$P_1A_1 - P_2A_2\cos\theta - F_x$ = Mass per sec × change of velocity

$= \rho Q$ (Final velocity in the direction of x − Initial velocity in the direction of x).

$$= \rho Q (V_2\cos\theta - V_1).$$

$$\boxed{F_x = P_1A_1 - P_2A_2\cos\theta + \rho Q (V_1 - V_2\cos\theta)}$$

The momentum equation in y-direction is given by:-

$$0 - P_2A_2\sin\theta - F_y = \rho Q (V_2\sin\theta - 0)$$

$$\boxed{F_y = -(\rho Q V_2\sin\theta + P_2A_2\sin\theta)}$$

THE MOMENTUM EQUATION :-

(I). The momentum equation is based on the law of conservation of momentum or momentum principle.

(II). The momentum principle states that the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction.

(III). The force acting on a fluid mass 'm' is given by newton's second law of motion:

$F = ma.$ a = Acceleration acting in the same direction as force F.

$F = m.\frac{dv}{dt}$ $a = \frac{dv}{dt}$

$= \frac{d(mv)}{dt}$

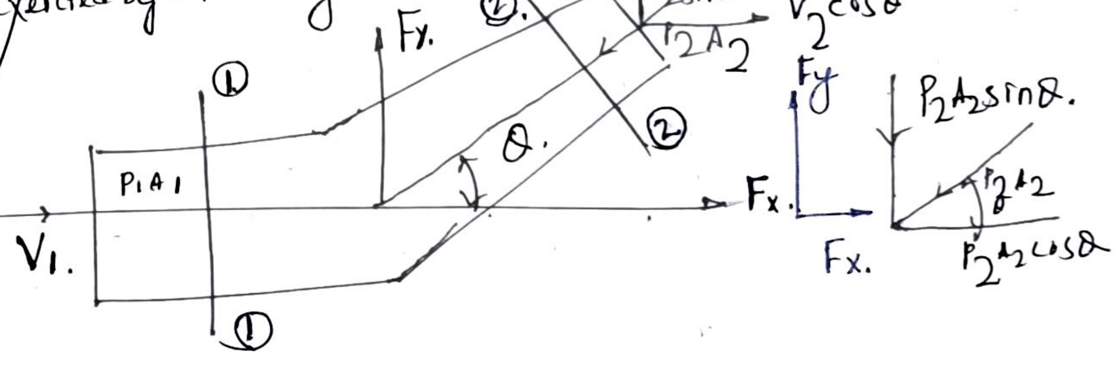
$$\boxed{F = \frac{d(mv)}{dt}}$$

This is known as momentum principle.

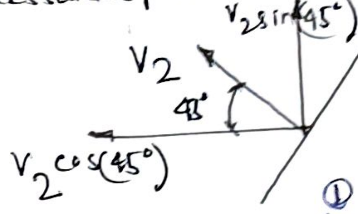
$$\boxed{F.dt = d(mv).}$$

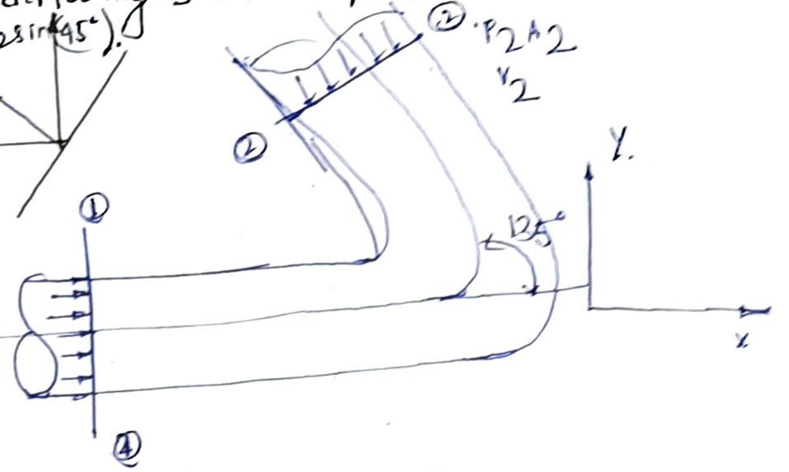
Which is known as the impulse momentum equation and states that the impulse of a force F acting on a fluid mass 'm' in short interval of time dt is equal to the change of momentum d(mv) in the direction of force.

The impulse-momentum equation is used to determine the resultant force exerted by a flowing fluid in the pipe bend.



2. 250 Litres/second of water is flowing in a pipe having a diameter of 300 mm. If the pipe is bent by 135° (that is change from initial to final direction), Find the magnitude and direction of resultant force on the bend. The pressure of water flowing is 39.24 N/cm².





$Q = 250$ Ltr/second

$= 0.25\ m^3/\text{second}$.

$\theta = 45°$

$P = 39.24 \times 10^4\ N/m^2$

$\rho = 1000\ kg/m^3$

$D = 300\ mm = 0.3\ m$

$A = \frac{\pi}{4} \times 0.3^2 = 0.0706\ m^2$

$V = \frac{Q}{A} = \frac{0.25}{0.0706}$

$= 3.541\ m/s$

$\boxed{V = 3.541\ m/s}$

$V_{1x} = 3.541\ m/s$

$V_{2x} = 3.541 \cos(45°)$

$= 2.503\ m/s$.

$P_{1x} = 39.24\ N/cm^2$

$P_{2x} = 39.24 \times \cos(45°)$

$= 27.746\ N/cm^2$.

$F_x = P_1 A_1 + P_{2x} A_2 \cancel{\cos(45°)} + \rho Q (V_{1x} - V_{2x})$.

$= 39.24 \times 10^4 \times 0.0706 + 27.746 \times 10^4 \times 0.0706 + 1000 \times 0.25 (3.541 - 2.503)$

$= 47.55 \times 10^3\ N$.

$\boxed{F_x = 47.55 \times 10^3\ N.}$

$V_{1y} = 0$

$V_{2y} = 3.541 \times \sin(45°)$

$= 2.503\ m/s$.

$P_{1y} = 0$

$P_{2y} = -39.24 \times \sin(45°)$

$= -27.746\ N/cm^2$

$F_y = \cancel{P_{1y} A_1}^{\,0} + P_{2y} A_2 + \rho Q (V_{1y} - V_{2y})$

$= -27.746 \times 10^4 \times 0.0706 + 1000 \times 0.25 (0 - 2.503)$

$= -18.962 \times 10^3\ N$. $\boxed{F_y = 18.962 \times 10^3\ N.}$

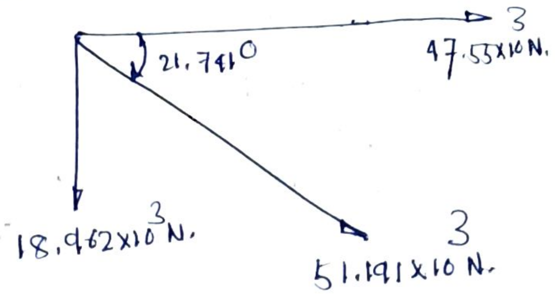
$8 \; C_d \tan(\theta/2)\sqrt{2g}\; H^{5/2}$.

$$F_R = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{(47.55\times10^3)^2 + (-18.962\times10^3)^2}$$

$$= 51.191\times10^3 \text{ N}$$

$$\boxed{F_R = 51.191\times10^3 \text{ N.}}$$



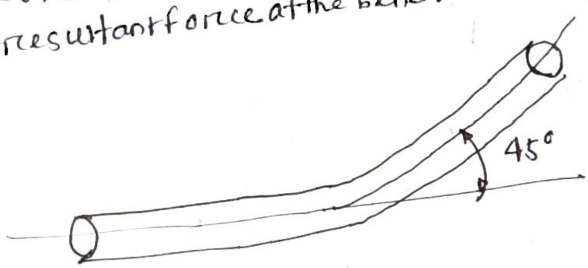
$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right).$$

$$= \tan^{-1}\left(\frac{-18.962\times10^3}{47.55\times10^3}\right)$$

$$= (21.741)^\circ$$

$$\boxed{\theta = (21.741)^\circ}$$

A 300mm diameter pipe carries water under a head of 20m with a velocity of 3.5 m/s. If the axis of the pipe turns through 45°. Find the magnitude and direction of resultant force at the bend.



$D = 300\text{mm} = 0.3\text{m}$.

$p = 1000\times9.81\times20$

$= 19.62\times10^4 \text{ N/m}^2$

$= 19.62 \text{ N/cm}^2$

$A = \frac{\pi}{4}\times0.3^2 = 0.0706 \text{ m}^2$

$V = 3.5 \text{m/s}$.

$\theta = 45°$

NOTCHES AND WEIRS

NOTCH :-

A notch is a device used for measuring the rate of flow of liquid through a small channel or a tank.

It may be defined as an opening in the side of a tank or a small channel in such a way ~~that~~ the liquid surface in the tank or channel is below the top edge of opening.

WEIR :-

A weir is a concrete or masonary structure, placed in an open channel over which flow occurs.

It is generally in the form of a vertical wall, with a sharp edge at the top running all the way across the open channel.

The notch is of small size and weir is of bigger size.

The notch is generally made of metallic plate while weir is made of concrete or masonary structure.

NAPPE OR VEIN :-

The sheet of water flowing through a notch or over a weir is called nappe or vein.

CREST OR SILL :-

The bottom edge of a notch or top of a weir over which water flows is known as the sill or crest.

1. According to shape of the opening.
   (i) Rectangular notch.
   (ii) Triangular notch.
   (iii) Trapezoidal notch
   (iv). Stepped notch.
2. According to the effects ~~on~~ of the sides on the nappe.
   (i) Notch with end contraction
   (ii) Notch without end contraction or suspended notch.

$\ldots 3/2 + \frac{8}{} C_d \tan(\theta/2)\sqrt{2g}\, H$

According to the shape of the opening :

(i) Rectangular weir
(ii) Trapezoidal weir / Cipolletti weir
(iii) Triangular weir

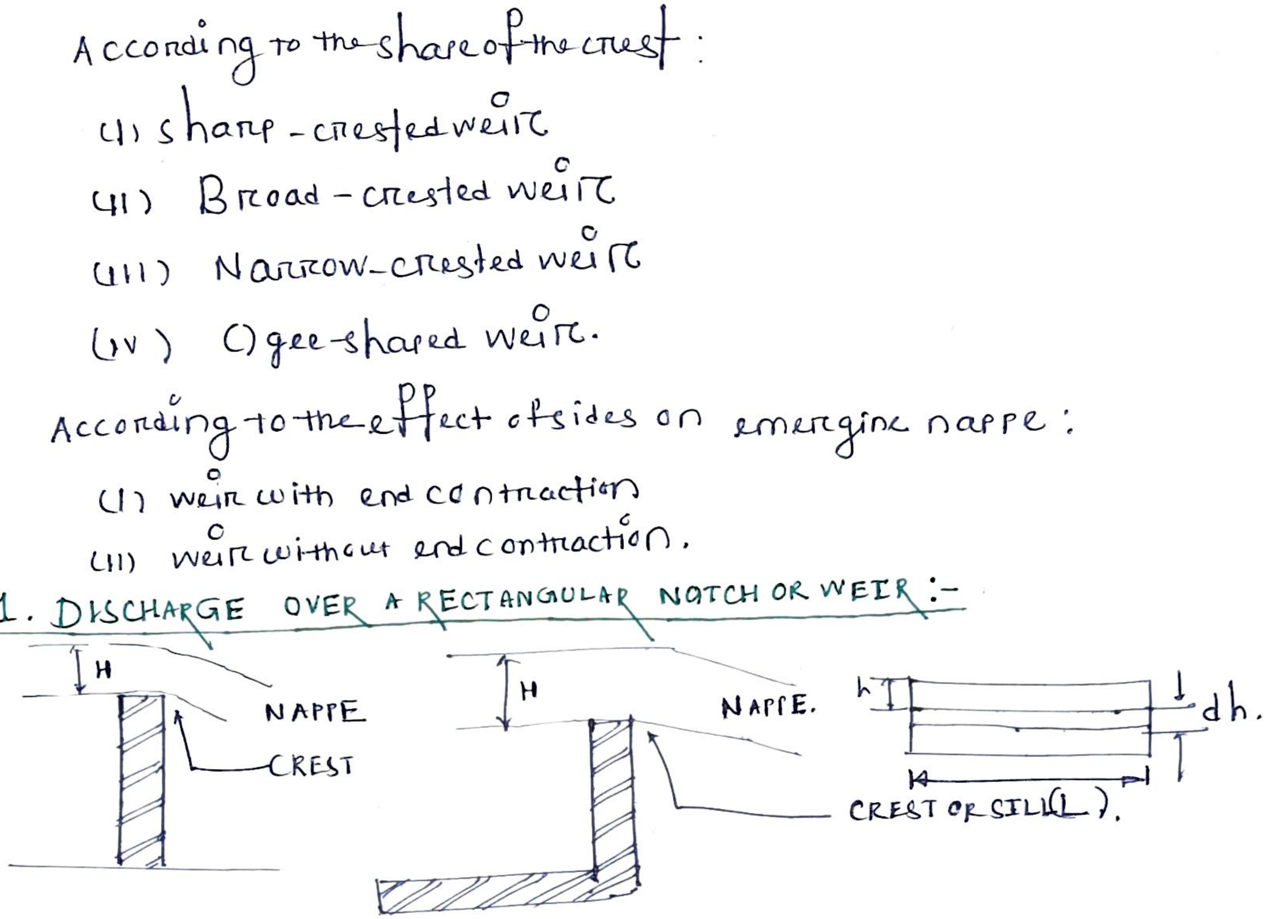
According to the shape of the crest :

(i) Sharp - crested weir
(ii) Broad - crested weir
(iii) Narrow-crested weir
(iv) Ogee-shaped weir.

According to the effect of sides on emerging nappe :

(i) weir with end contraction
(ii) weir without end contraction.

1. DISCHARGE OVER A RECTANGULAR NOTCH OR WEIR :-



The discharge over a rectangular weir or notch is the same.

$$\boxed{\text{Discharge } Q = \int_0^H C_d \times \text{Area of the strip} \times \text{Theoritical velocity}}$$

$$= \int_0^H C_d \times (L \times dh) \times \sqrt{2gh}$$

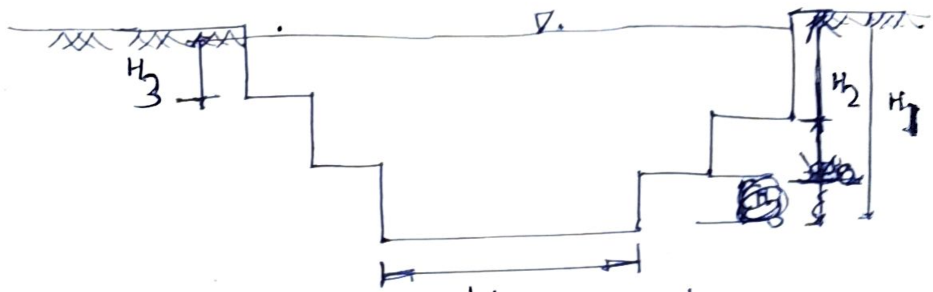
$$= \int_0^H C_d\, L\, \sqrt{2g}\, (\sqrt{h}\,.\,dh).$$

$$\ldots ^{3/2} + \frac{8}{15} C_d \tan(\theta/2)\sqrt{2g}\, H$$

4. DISCHARGE OVER A STEPPED NOTCH :-

A stepped notch is a combination of rectangular notches.

The discharge through stepped notch is equal to the sum of the discharges through different rectangular notches.



$$Q = Q_1 + Q_2 + Q_3$$

$$= \frac{2}{3} C_d L_1 \sqrt{2g}\left[H_1^{3/2} - H_2^{3/2}\right] + \frac{2}{3} C_d L_2 \sqrt{2g}\left[H_2^{3/2} - H_3^{3/2}\right] + \frac{2}{3} C_d L_3 \sqrt{2g}\left[H_3^{3/2}\right]$$

EFFECT ON DISCHARGE OVER A NOTCH OR WEIR DUE TO ERROR IN THE MEASUREMENT OF HEAD :-

1. FOR RECTANGULAR WEIR :

$$\boxed{Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}.}$$

$$= K H^{3/2} \qquad K = \frac{2}{3} C_d L \sqrt{2g}.$$

$$\frac{dQ}{dH} = K \cdot \frac{3}{2} H^{3/2 - 1}$$

$$= K \cdot \frac{3}{2} H^{1/2}.$$

$$\boxed{dQ = \frac{3K}{2} H^{1/2}.\, dH.}$$

$$\frac{dQ}{Q} = \frac{K \frac{3}{2} . H^{1/2}\, dH}{K\, H^{3/2}.}$$

$$= \frac{3}{2} \frac{dH}{H.}$$

$$\boxed{\frac{dQ}{Q} = \frac{3}{2} \frac{dH}{H.}}$$

An error of 1% in measuring H will produce 1.5% error in discharge over a rectangular weir.

A triangular notch or weir is preferred to a rectangular weir or notch.

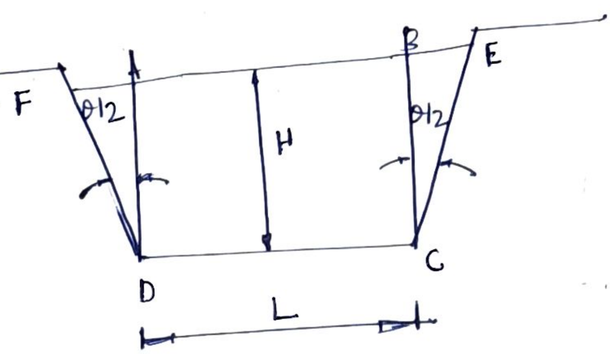
The expression for discharge for a right angled v-notch or weir is very simple.

For measuring low discharge a triangular notch gives more accurate results than a rectangular notch.

In case of triangular notch only one reading i.e. H is required for the computation of discharge.

Ventilation of a triangular notch is not necessary.

DISCHARGE OVER A TRAPEZOIDAL NOTCH OR WEIR:-



H = Height of water over the notch

L = Length of the crest of the notch.

Discharge through rectangular portion: $Q_1 = \frac{2}{3} \times C_{d_1} \times L \sqrt{2g} \times H^{3/2}$

Triangular portion $Q_2 = \frac{8}{15} \times C_{d_2} \sqrt{2g}\, H^{5/2} \tan(\theta/2)$.

$Q = Q_1 + Q_2$

$$\boxed{Q = \frac{2}{3} C_{d_1} \times L \sqrt{2g} \times H^{3/2} + \frac{8}{15} C_{d_2} \sqrt{2g}\, H^{5/2} \tan(\theta/2)}$$

Discharge dQ = $C_d \times$ Area of the strip $\times$ Velocity

$$= C_d\, 2(H-h)\tan(\theta/2) \times \sqrt{2gh}\, dh.$$

$$= C_d\, 2\tan\theta/2 \sqrt{2g} \quad \sqrt{h}\cdot(H-h)\, dh.$$

$$Q = \int_0^H C_d\, 2\tan(\theta/2)\sqrt{2g} \quad \sqrt{h}\,(H-h)\, dh.$$

$$= C_d\, 2\tan(\theta/2)\sqrt{2g} \int_0^H \sqrt{h}\,(H-h)\, dh.$$

$$= C_d\, 2\tan(\theta/2)\sqrt{2g} \int_0^H \left[H\sqrt{h} - h^{3/2}\right] dh$$

$$= C_d\, 2\tan(\theta/2)\sqrt{2g} \left[H\frac{h^{1/2+1}}{1/2+1} - \frac{h^{3/2+1}}{3/2+1}\right]_0^H$$

$$= C_d\, 2\tan(\theta/2)\sqrt{2g} \left[\frac{H H^{3/2}}{3/2} - \frac{H^{5/2}}{5/2}\right]$$

$$= C_d\, 2\tan(\theta/2)\sqrt{2g}. \left[\frac{2H^{5/2}}{3} - \frac{2H^{5/2}}{5}.\right]$$

$$= C_d\, 2\tan(\theta/2)\sqrt{2g}. \left[\frac{10H^{5/2} - 6H^{5/2}}{15}\right]$$

$$= C_d\, 2\tan(\theta/2)\sqrt{2g}\left(\frac{4H^{5/2}}{15}\right).$$

$$= \frac{8}{15} C_d \tan(\theta/2)\sqrt{2g}\; H^{5/2}$$

$$\boxed{Q = \frac{8}{15} C_d \tan\left(\frac{\theta}{2}\right)\sqrt{2g}\; H^{5/2}}$$

$$= \int_0^H C_d L\sqrt{2g}\int_0^H \sqrt{h}\,dh.$$

$$= C_d\sqrt{2g}\,L\left[\frac{h^{1/2+1}}{1/2+1}\right]_0^H$$

$$= \frac{2}{3} C_d\sqrt{2g}\times L\, H^{3/2}.$$

$$\boxed{Q = \frac{2}{3} C_d.\sqrt{2g}\times L\times H^{3/2}}$$

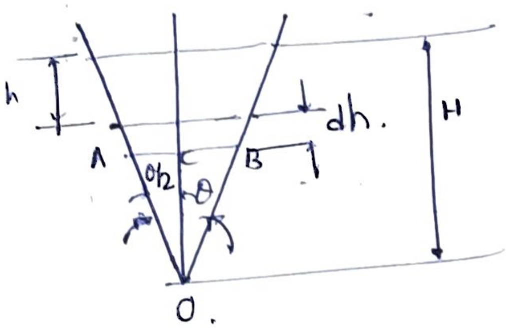
C_d = coefficient of discharge

H = Head of water over crest

L = Length of the notch or weir.

2. DISCHARGE OVER A TRIANGULAR NOTCH OR WEIR :-

The expression for discharge over a triangular notch or weir is same.



$$\tan\left(\frac{\theta}{2}\right) = \frac{AC}{OC}.$$

$$AC = OC\tan\left(\frac{\theta}{2}\right).$$

$$AB = 2\times AC$$

$OC = H-h$

$$= 2\times OC\tan\left(\frac{\theta}{2}\right).$$

$$= 2(H-h)\tan\left(\frac{\theta}{2}\right).$$

$$\boxed{L = 2(H-h)\tan\left(\frac{\theta}{2}\right)}$$

Area of the strip = $L\times dh$ = $2(H-h)\tan(\theta/2)\,dh.$

$$\boxed{A = 2(H-h)\tan(\theta/2)\,dh.}$$

FOR TRIANGULAR WEIR

$$Q = \frac{8}{15} C_d \tan(\theta/2) \times H^{5/2}$$

$$K = \frac{8}{15} C_d \tan(\theta/2).$$

$$= K H^{5/2}.$$

$$\frac{dQ}{dH} = K \cdot \frac{5}{2} H^{\frac{5}{2}-1}.$$

$$= \frac{5}{2} K \cdot H^{3/2}.$$

$$dQ = \frac{5K}{2} H^{3/2} dH.$$

$$\frac{dQ}{Q} = \frac{\frac{5}{2} \cdot \left(\frac{8}{15} C_d \tan(\theta/2) H^{3/2}\right) dH}{\left(\frac{8}{15} C_d \tan(\theta/2) . H^{5/2}\right)}$$

$$\boxed{\frac{dQ}{Q} = \frac{dH}{H} \cdot \frac{5}{2}}$$

An error of 1% in measuring H will produce 2.5% error in discharge over a triangular weir or notch.

(1) A rectangular notch 40cm long is used for measuring a discharge of 30 Ltrs/second. An error of 1.5mm was made, while measuring the head over the notch. Calculate the percentage error in the discharge. Take $C_d = 0.62$.

$$L = 40cm$$
$$= 0.4M.$$
$$Q = 30 \text{ Ltr/sec.}$$
$$= 0.03 \, m^3/sec.$$

$$dH = 1.5mm = 0.0015m.$$

$$C_d = 0.62.$$

H = Height of water over rectangular notch.

$$Q = \frac{2}{3} C_d L \sqrt{2g} \times H^{3/2}.$$

$$0.03 = \frac{2}{3} \times 0.62 \times 0.4 \sqrt{2 \times 9.81} \times H^{3/2}.$$

$$H = \left[\frac{0.03}{2/3 \times 0.62 \times 0.4 \times \sqrt{2 \times 9.81}} \right]^{2/3}.$$

$$= \cancel{0.118m} \; 0.1214 \, m.$$

$$\boxed{H = \cancel{0.118m.} \; 0.1214m.}$$

$$\frac{dQ}{Q} = \frac{3}{2} \cdot \frac{dH}{H}$$

$$= \frac{3}{2} \times \frac{1.5 \times 10^{-3}}{0.\cancel{118}1214}$$

$$= 0.0185$$

$$= 1.85\%$$

$$\boxed{\frac{dQ}{Q} = 1.85\%}$$

2. A rightangled V-notch is used for measuring a discharge of 30 litr/sec. An error of 1.5 mm was made while measuring the head over the notch. Calculate the percentage error in the discharge. Take $C_d = 0.62$.

$Q = 0.03\ m^3/sec$ $\quad \theta = 90^\circ$

$C_d = 0.62$.

$dH = 1.5\ mm.$

$= 1.5 \times 10^{-3}\ m.$

$$Q = \frac{8}{15} C_d \times \tan(\theta/2) \times \sqrt{2g}\ H^{5/2}.$$

$$H = \left[\frac{0.03}{\frac{8}{15} \times 0.62 \times \tan(45^\circ) \times \sqrt{2 \times 9.81}}\right]^{2/5}$$

$= 0.211\ m$

$$\boxed{H = 0.211\ m.}$$

$$\frac{dQ}{Q} = \frac{5}{2} \times \frac{1.5 \times 10^{-3}}{0.211}$$

$= 0.0177$

$= 1.77\%$

$$\boxed{\frac{dQ}{Q} = 1.77\%}$$

3. The head of water over a triangular notch of angle 60° is 50 cm. and coefficient of discharge is 0.62. The flow measured by it is to be within an accuracy of 1.5% up or down. Find the limiting values of the head.

$\theta = 60^\circ$

$H = 50\ cm = 0.5\ m.$

$C_d = 0.62.$

$$Q = \frac{8}{15} \times C_d \sqrt{2g} \times \tan(\theta/2)\ H^{5/2}$$

$$Q = \frac{8}{15} \times 0.62 \times \sqrt{2 \times 9.81} \times \tan(60^\circ/2) \times 0.5^{5/2}$$

$= 0.\cancel{1868}\ 1494\ m^3/sec$

$$\boxed{Q = \cancel{0.1868}\ 0.1494\ m^3/sec.}$$

$$\frac{dQ}{Q} = 2.5 \times \frac{dH}{H}$$

$$\frac{dQ}{Q} = 1.5\%$$

$$= 0.015$$

$$0.015 = 2.5 \frac{dH}{H}$$

$$dH = \frac{0.015}{2.5} \times H$$

$$= \frac{0.015}{2.5} \times 0.5$$

$$= \cancel{0.3m} \; 0.003m$$

$$\boxed{dH = \cancel{0.3m} \; 0.003M}$$

The limiting values of head : $H \pm dH$

0.5 ± 0.003

$$H \begin{cases} 0.503m. \text{ or } 0.497m \\ 503mm \text{ or } 497mm. \end{cases}$$

$\sqrt{\quad} H^{3/2} + \frac{8}{\cdot} C_d$ ...

Flow through a right-angled weir

$\theta = 90^\circ$

$C_d = 0.6$

$H = 360\ mm$

$= 0.36\ m.$

$$Q = \frac{8}{15} C_d \tan(\theta/2) \sqrt{2g}\, H^{5/2}$$

$$= \frac{8}{15} \times 0.6 \times \tan(45^\circ) \times \sqrt{2 \times 9.81} \times 0.36^{5/2}$$

$= 0.1102\ m^3/sec.$

$$\boxed{Q = 0.1102\ m^3/sec}$$

For rectangular weir:-

$L = 1m.$

$C_d = 0.7.$

$$Q = \frac{2}{3} C_d L \sqrt{2g}\, H^{3/2}.$$

$$H = \left[\frac{0.1102}{2/3 \times 0.7 \times 1 \times \sqrt{2 \times 9.81} \times}\right]^{2/3}$$

$= 0.14166\ mm$

$$\boxed{H = 141.65\ mm.}$$

5. Water flows over a rectangular weir 1 m wide at a depth of 150 mm and afterwards passes through a triangular right-angled weir. Take C_d for rectangular and triangular weir as 0.62 and 0.59 respectively. Find the depth over the triangular weir.

$L = 1\,m$

$H = 150\,mm = 0.15\,m.$

$C_d = 0.62.$

$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g}\; H^{3/2}$$

$$= \frac{2}{3} \times 0.62 \times 1 \times \sqrt{2 \times 9.81} \times (0.15)^{3/2}$$

$$= 0.1063\ m^3/sec.$$

$C_d = 0.59$ (For triangular weir)

$$Q = \frac{8}{15} C_d \tan(\theta/2) \sqrt{2g}\, H^{5/2}.$$

For right angled weir
$\theta = 90°$

$$0.1063 = \frac{8}{15} \times 0.59 \times \tan(45°) \times \sqrt{2 \times 9.81} \times H^{5/2}$$

$$H = \left[\frac{0.1063}{\frac{8}{15} \times 0.59 \times (\tan 45°) \times \sqrt{2 \times 9.81}}\right]^{2/5}$$

$= 0.3572\ m$

$$\boxed{H = 0.3572\,m \text{ or } 357\,mm.}$$

6. Water flows through a triangular right-angled weir first and then over a rectangular weir of 1 m width. The discharge coefficients of the triangular and rectangular weirs are 0.6 and 0.7 respectively. If the depth of water over triangular weir is 360 mm. Find the depth of water over the rectangular weir.

③. The head of water over a rectangular notch is 900mm. The discharge is 300 Ltres/sec. Find the length of the notch when $C_d = 0.62$.

$H = 900\,mm$
$= 0.9\,m.$

$Q = 300\,ltr/sec.$
$= 0.3\,m^3/sec$

$C_d = 0.62$

$$Q = \frac{2}{3} C_d L \sqrt{2g}\, H^{3/2}$$

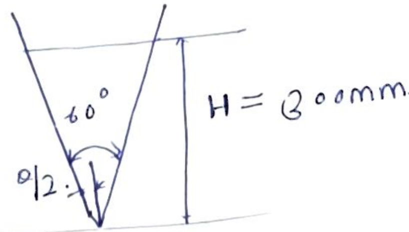
$$L = \frac{0.3}{\left(2/3 \times 0.62 \sqrt{2 \times 9.81} \times 0.9^{3/2}\right)}.$$

$= 0.19819 \simeq 0.192\,m.$

$$\boxed{L = 192\,mm}$$

④ Find the discharge over a triangular notch of angle 60° when the head over the v-notch is 0.3 m. $C_d = 0.6$.

$\theta = 60°$
$H = 0.3\,M$
$C_d = 0.6$



$$Q = \frac{8}{15} C_d \times \sqrt{2g} \times \tan\theta/2 \times H^{5/2}$$

$$= \frac{8}{15} \times 0.60 \times \sqrt{2 \times 9.81} \times \tan(30°) \times 0.3^{5/2}$$

$= 0.0403\,m^3/sec$

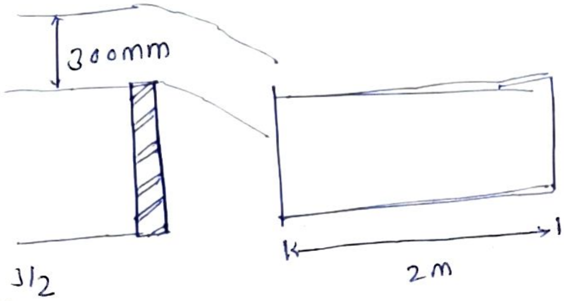
$$\boxed{Q = 0.0403\,m^3/sec. \text{ or } Q = 40.3\,Ltr/sec.}$$

(1) Find the discharge of water flowing over a rectangular notch of 2m length when the constant head over the notch is 300mm. Take $C_d = 0.6$

$L = 2m.$

$H = 0.3m.$

$C_d = 0.60$



$$Q = \frac{2}{3} C_d \sqrt{2g}\, L\, H^{3/2}$$

$$= \frac{2}{3} \times 0.60 \times \sqrt{2 \times 9.81} \times 2 \times 0.3^{3/2}$$

$$= 0.582 m^3/sec.$$

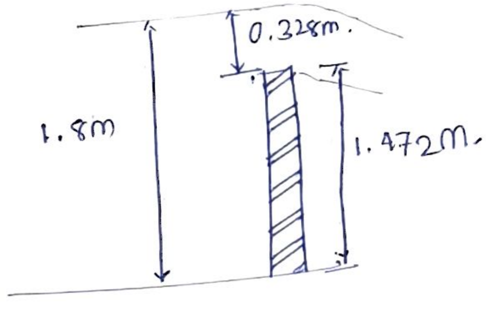
$$\boxed{Q = 0.582 m^3/sec.}$$

(2) Determine the height of a rectangular weir of length 6m to built across a rectangular channel. The maximum depth of water on the upstream side of the weir is 1.8m and discharge is 2000 Ltr/sec. $C_d = 0.6$.

$L = 6m.$

$Q = 2000 Ltr/sec = 2 m^3/sec.$

$C_d = 0.6$



$$Q = \frac{2}{3} C_d \sqrt{2g}\, L\, H^{3/2}.$$

$$2 = \frac{2}{3} \times 0.6 \times \sqrt{2 \times 9.81} \times 6 \times H^{3/2}$$

$$H^{3/2} = 0.1881$$

$$H = 0.1881^{2/3} = 0.328m.$$

$$\boxed{H = 0.328m}$$

Height of weir $= 1.8 - 0.328$

$= 1.472m.$

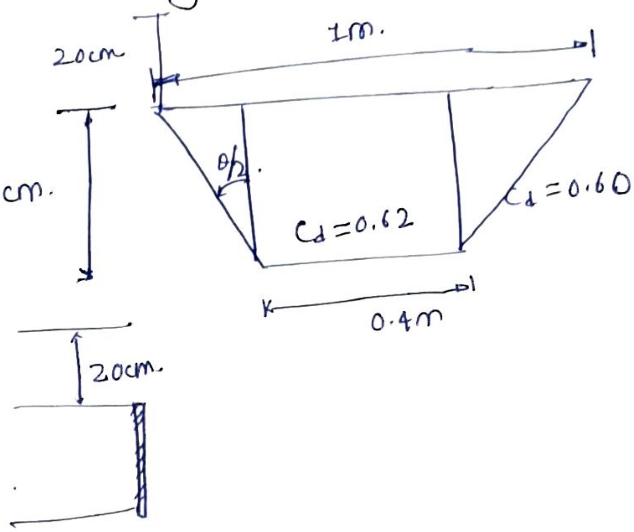
7. Find the discharge through a trapezoidal notch which is 1 m wide at the top and 0.4 m at the bottom and is 30 cm height. The head of water on the notch is 20 cm. Assume C_d for rectangular portion 0.62 and triangular portion 0.60.

$\tan(\theta/2) = \frac{0.3}{0.3}$

~~$\theta/2 = \tan^{-1}(1)$~~

$=$

$\boxed{\tan(\theta/2) = 1}$



$$Q = \frac{2}{3} C_d \sqrt{2g}\, L (H)^{3/2} + \frac{8}{15} C_d \cdot \sqrt{2g} \cdot \tan(\theta/2) H^{5/2}.$$

$$= \frac{2}{3} \times 0.62 \times \sqrt{2 \times 9.81} \times 0.4 \times 0.2^{3/2} + \frac{8}{15} \times 0.60 \times \sqrt{2 \times 9.81} \times 1 \times 0.2^{5/2}$$

$= 0.09085 m^3/sec.$

$= 90.85$ Ltrs/sec.

$\boxed{Q = 90.85 \text{ Ltrs/sec.}}$

8. Find the discharge through the notch if $C_d = 0.62$ for all sections.

$H_1 = 50 + 30 + 15$

$= 95$ cm

$= 0.95$ m.

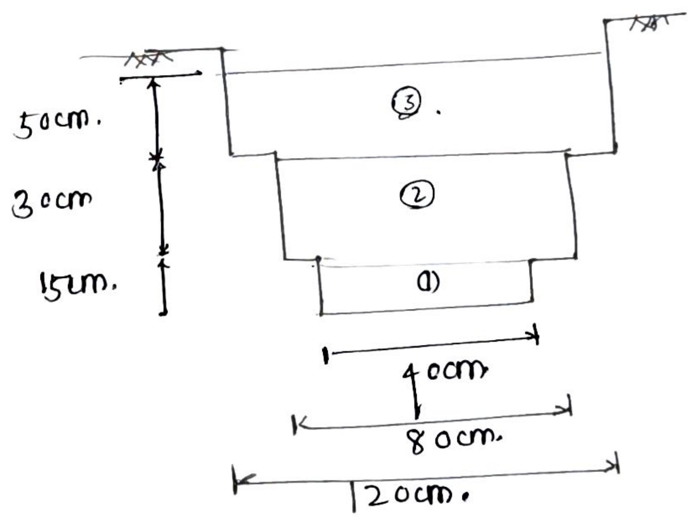
$H_2 = 0.8$ m

$H_3 = 0.5$ m,

$L_1 = 0.4$ m

$L_2 = 0.8$ m

$L_3 = 1.2$ m.



$\quad H^{3/2} + \frac{8}{15} C_d \tan(\theta/2)\sqrt{2g}\, H^{5/2}$

$Q = Q_1 + Q_2 + Q_3$

$Q_1 = \frac{2}{3} C_d L_1 \sqrt{2g} \left[H_1^{3/2} - H_2^{3/2} \right]$

$= \frac{2}{3} \times 0.62 \times \sqrt{2 \times 9.81} \left[0.95^{3/2} - 0.8^{3/2} \right] \times 0.4$

$= 0.154\, m^3/sec$ $\quad \boxed{Q_1 = 0.154\, m^3/sec.}$

$Q_2 = \frac{2}{3} C_d L_2 \sqrt{2g} \left[H_2^{3/2} - H_3^{3/2} \right]$

$= \frac{2}{3} \times 0.62 \times 0.8 \times \sqrt{2 \times 9.81} \left[0.8^{3/2} - 0.5^{3/2} \right]$

$= 0.530\, m^3/sec$

$\boxed{Q_2 = 0.530\, m^3/sec.}$

$Q_3 = \frac{2}{3} C_d . L_3 \sqrt{2g} \times H_3^{3/2}$

$= \frac{2}{3} \times 0.62 \times 1.2 \times \sqrt{2 \times 9.81} \times 0.5^{3/2}$

$= 0.776\, m^3/sec.$

$\boxed{Q_3 = 0.776\, m^3/sec.}$

$Q = Q_1 + Q_2 + Q_3$

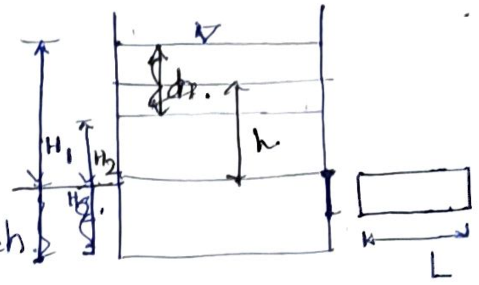
$= 0.154 + 0.530 + 0.776$

$= 1.460\, m^3/sec$

$\boxed{Q = 1.460\, m^3/sec}$

TIME REQUIRED TO EMPTY A RESERVOIR OR A TANK WITH A RECTANGULAR WEIR OR NOTCH:-

A reservoir or tank with uniform C/A area A. A rectangular weir or notch is provided in one of its sides.



L = Length of crest of the weir or notch.

C_d = coefficient of discharge

H_1 = initial height of the liquid above the crest of notch.

H_2 = Final height of the liquid above the crest of notch.

T = Time required in seconds to lower the height of liquid from H_1 to H_2.

$$\boxed{Q = \frac{2}{3} C_d L \sqrt{2g}\, h^{3/2}.}$$

$$Q\,dT = -A\,dh.$$

$$-A\,dh = \left(\frac{2}{3} C_d L \sqrt{2g}\, h^{3/2}\right) dT$$

$$\int_0^T dT = \int_{H_1}^{H_2} \frac{-A\,dh}{\frac{2}{3} C_d L \sqrt{2g}\, h^{3/2}}.$$

$$T = \frac{-A}{\frac{2}{3} C_d L \sqrt{2g}} \int_{H_1}^{H_2} -\frac{dh}{h^{3/2}}.$$

$$= -\frac{A}{\frac{2}{3} C_d L \sqrt{2g}} \int_{H_1}^{H_2} -h^{3/2}\,dh.$$

$$= \frac{-A}{\frac{2}{3} C_d L \sqrt{2g}} \left[\frac{h^{-3/2+1}}{-\frac{3}{2}+1} \right]_{H_1}^{H_2}.$$

$$= -\frac{A}{\frac{2}{3} C_d L \sqrt{2g}} \left[\frac{-\cancel{1/2}}{\sqrt{h}} \right]_{H_1}^{H_2} = \frac{3A}{C_d L \sqrt{2g}} \left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}} \right]$$

$$\boxed{T = \frac{3A}{C_d L\sqrt{2g}}\left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}}\right].}$$

TIME REQUIRED TO EMPTY A RESERVOIR OR A TANK WITH A TRIANGULAR WEIR OR NOTCH :—

A Reservoir or tank with uniform c/s area A, having a triangular weir or notch in one of its sides.

θ = Angle of notch.

C_d = coefficient of discharge

H_1 = Initial height of liquid above apex of notch.

H_2 = Final height of liquid above the apex of notch.

T = Time required in seconds to lower the height from H_1 to H_2 above the apex of the notch.

$$-A\,dh = Q\,dT.$$

$$Q = \frac{8}{15} C_d \tan(\theta/2) \times \sqrt{2g}\, h^{5/2}.$$

$$dT = \frac{-A\,dh}{\frac{8}{15} C_d \tan(\theta/2)\sqrt{2g}\, h^{5/2}}.$$

$$\int_0^T dT = \int_{H_1}^{H_2} \frac{-A\,dh}{\frac{8}{15} C_d \tan(\theta/2)\sqrt{2g} \times h^{5/2}}$$

$$T = \frac{-15A}{8C_d \tan(\theta/2)\sqrt{2g}} \int_{H_1}^{H_2} \frac{dh}{h^{5/2}}$$

$$= \frac{-15A}{8C_d \tan(\theta/2)\sqrt{2g}} \int_{H_1}^{H_2} h^{-5/2}\,dh.$$

$$= \frac{-15A}{8C_d \tan(\theta/2)\sqrt{2g}} \left[\frac{h^{-5/2+1}}{-\frac{5}{2}+1}\right]_{H_1}^{H_2}$$

$$= \frac{-15A}{8C_d\sqrt{2g}\tan(\theta/2)} \times \left(-\frac{2}{3}\right) \left[\frac{1}{h^{3/2}}\right]_{H_1}^{H_2}$$

$$= \frac{5A}{4C_d \tan(\theta/2)\sqrt{2g}}\left[\frac{1}{H_2^{3/2}} - \frac{1}{H_1^{3/2}}\right]$$

$$\boxed{T = \frac{5A}{4C_d \tan(\theta/2)\sqrt{2g}}\left[\frac{1}{H_2^{3/2}} - \frac{1}{H_1^{3/2}}\right]}$$

$$= C_d L \sqrt{2g} \sqrt{\frac{4}{27}} \; H^{3/2}.$$

$$= 1.705 \, C_d L \sqrt{2g} \, H^{3/2}$$

$$\boxed{Q = 1.705 \, C_d L \sqrt{2g} \, H^{3/2}.}$$

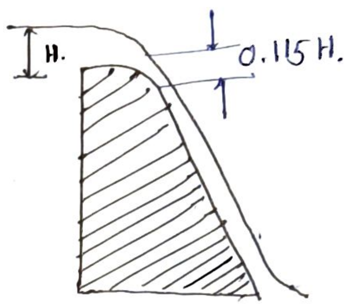
DISCHARGE OVER A NARROW CRESTED WEIR:

$$\boxed{Q = \frac{2}{3} C_d L \sqrt{2g} \, H^{3/2}.}$$

FOR $2L < H$.
Similar to rectangular weir or notch.

DISCHARGE OVER AN OGEE WEIR:

The crest of weir rises upto maximum height of 0.115 H.



$$\boxed{Q = \frac{2}{3} C_d L \sqrt{2g} \, H^{3/2}.}$$

H = Height of water above inlet of weir.

$$\frac{V^2}{2g} = H-h.$$

$$V = \sqrt{2g(H-h)}$$

$$\boxed{V = \sqrt{2g(H-h)}}$$

The discharge over weir : $Q = C_d \times$ Area of flow $\times$ velocity

$$= C_d \times L \cdot h \times \sqrt{2g(H-h)}.$$

$$Q = C_d L \sqrt{2g(H-h)h^2}$$

$$= C_d L \sqrt{2g(Hh^2 - h^3)}$$

$$\boxed{Q = C_d L \sqrt{2g(Hh^2 - h^3)}}$$

$$\frac{d}{dh}(Hh^2 - h^3) = 0$$

$$H(2h) - 3h^2 = 0$$

$$3h^2 = 2Hh.$$

$$\boxed{H = \frac{3h}{2}}$$

$$\boxed{h = \frac{2H}{3}}$$

$$Q_{max^m} = C_d L \sqrt{2g} \sqrt{H\left(\frac{2H}{3}\right)^2 - \left(\frac{2H}{3}\right)^3}$$

$$= C_d L \sqrt{2g} \sqrt{\frac{4H^3}{9} - \frac{8H^3}{27}} = C_d L \sqrt{2g} \sqrt{\frac{12H^3 - 8H^3}{27}}$$

$$= C_d L \sqrt{2g} \sqrt{\frac{4H^3}{27}}$$

$$\frac{8}{15} C_d \sqrt{2g} \tan(\theta/2)\, H^{5/2} = \frac{2}{15} C_d \sqrt{2g}\, H^{5/2}$$

$$\tan(\theta/2) = \frac{2}{15} \times \frac{15}{8}$$

$$= \frac{1}{4}.$$

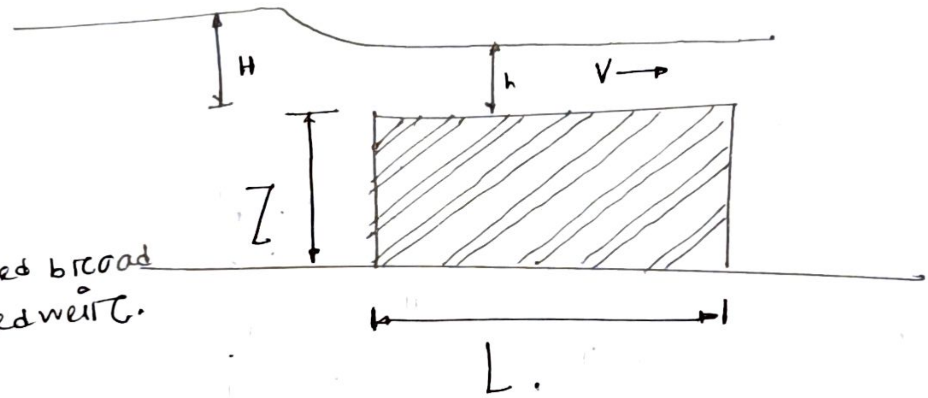
$$\boxed{\tan(\theta/2) = \frac{1}{4}}$$

Discharge through cippoletti weir $\boxed{Q = \frac{2}{3} C_d L \times \sqrt{2g} \times [H]^{3/2}.}$

A weir having a wide crest is known as broad-crested weir

H = Height of water above crest.

L = Length of the crest.



(i) $2L > H$

The weir is called broad crested weir.

(ii) $2L < H$

The weir is called narrow crested weir.

h = head of water at the middle of the weir.

V = velocity of flow over the weir.

Apply Bernoulli's equation to the still water surface on the upstream side and running water at the end of weir.

$$\cancel{\frac{P_1}{\rho g}}^{0} + \cancel{\frac{V_1^2}{2g}}^{0} + H = \cancel{\frac{P_2}{\rho g}}^{0} + \frac{V_2^2}{2g} + h.$$

VELOCITY OF APPROACH :

Velocity of approach is defined as the velocity with which the water approaches or reaches the weir or notch before it flows over it.

Additional head due to velocity of approach (ha) : $\frac{V_a^2}{2g}$.

$$\boxed{h_a = \frac{V_a^2}{2g}}$$

$$V_a = \frac{Q}{\text{Area of channel}}$$

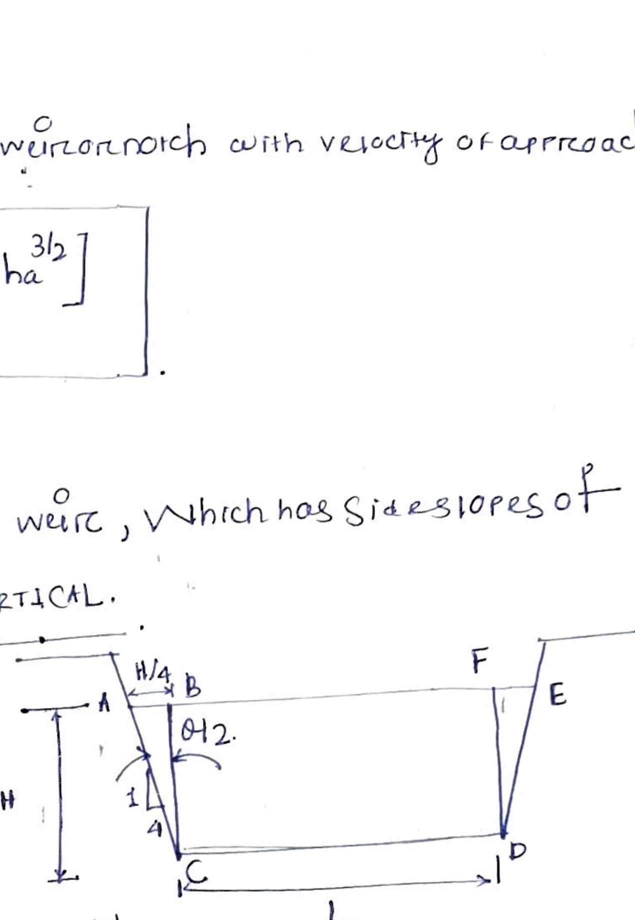
Discharge over a rectangular weir or notch with velocity of approach :-

$$\boxed{Q = \frac{2}{3} C_d L \times \sqrt{2g} \left[(H_1 + h_a)^{3/2} - h_a^{3/2} \right]}$$

CIPOLLETTI WEIR OR NOTCH :-

Cipolletti weir is a trapezoidal weir, which has side slopes of

$\boxed{1H : 4V}$ 1 HORIZONTAL : 4 VERTICAL.



discharge through rectangular weir with end contractions :-

$$Q = \frac{2}{3} C_d \sqrt{2g} \, (L - 0.2H) H^{3/2}$$

$$= \frac{2}{3} C_d \sqrt{2g} \, L H^{3/2} - \frac{2}{15} C_d \sqrt{2g} \, L H^{5/2}$$

discharge through triangular notch.

$$\Rightarrow Q = \frac{8}{15} C_d \sqrt{2g} \tan(\theta/2) H^{5/2}.$$

$$\tan(\theta/2) = \frac{AB}{BC}$$

$$= \frac{H/4}{H}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{1}{4} = 0.25$$

$$\frac{\theta}{2} = \tan^{-1}(0.25)$$

$$= 14^\circ 2'$$

$$\boxed{\theta = 28^\circ 4'}$$

Find the time required to lower the water level from 3m to 2m in a reservoir of dimension 80m × 80m by a rectangular notch of length 1.5m. $C_d = 0.62$.

$H_1 = 3m$

$H_2 = 2m$.

$A = 80 \times 80 = 6400 m^2$

$L = 1.5 m$

$C_d = 0.62$.

$$T = \frac{3A}{C_d \sqrt{2g} L}\left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}}\right]$$

$$= \frac{3 \times 6400}{0.62 \times \sqrt{2 \times 9.81} \times 1.5}\left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}\right]$$

$= 604.78$ sec. $= 10$ min. 4.78 sec.

$= 10$ min. 5 sec.

$$\boxed{T = 10 \text{ min. } 5 \text{ sec.}}$$

Find the time required to lower the water level from 3m to 2m in a reservoir of dimension 80m × 80m by a right angled V-notch of height 1.5m. $C_d = 0.62$

$H_1 = 3m$ $\qquad \theta = 90°$

$H_2 = 2m$.

$A = 6400 m^2$

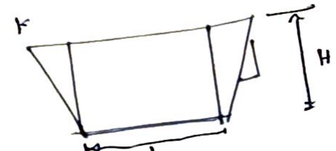
$C_d = 0.62$.

$$T = \frac{5A}{4 C_d \sqrt{2g} \tan(\theta/2)}\left[\frac{1}{H_2^{3/2}} - \frac{1}{H_1^{3/2}}\right]$$

$$= \frac{5 \times 6400}{4 \times 0.62 \times \sqrt{2 \times 9.81} \times (\tan 45°)}\left[\frac{1}{2^{3/2}} - \frac{1}{3^{3/2}}\right] = 7 \text{ min. } 49.3 \text{ sec.}$$

$= 7$ min. 50 sec.

$$\boxed{T = 7 \text{ min. } 50 \text{ sec.}}$$



$— \ 3/2 \quad \perp \ 8 \quad C_d \tan(\theta/2)\sqrt{2g}\ H^{5/2}.$

A rightangled V-notch is inserted in the side of a tank of length 4m and width 2.5m. Initial height of water above the apex of the notch is 30cm. Find the height of water above apex if the time required to lower the head of tank from 30cm height is 3 minutes. $C_d = 0.60$.

$L = \frac{\text{Tank}}{4m}$ $\quad \theta = 90^\circ$

$w = 2.5m.$ $\quad A = 4 \times 2.5$

$= 10m^2$

$H_1 = 0.3\,m$

$H_2 = ?$

$T = 3$ mins

$= 3 \times 60$

$= 180$ sec.

$C_d = 0.6$

$\cancel{T = \frac{5A}{4 C_d \sqrt{2g}\,\tan 45^\circ}\left[\frac{1}{H_2^{3/2}} - \frac{1}{H_1^{3/2}}\right]}$

$$T = \frac{5A}{4 C_d \sqrt{2g}\tan(\theta/2)}\left[\frac{1}{H_2^{3/2}} - \frac{1}{H_1^{3/2}}\right]$$

$$\frac{180 \times 0.62 \times \sqrt{2 \times 9.81} \times 4 \times \tan 45^\circ}{5 \times 10} = \left[\frac{1}{H_2^{3/2}} - \frac{1}{0.3^{3/2}}\right]$$

$\cancel{39.546 = \left[\frac{1}{0.3^{3/2}} - \frac{1}{H_2^{3/2}}\right]}$

$\cancel{\frac{1}{H_2^{3/2}} = \frac{1}{0.3^{3/2}} - 39.546.}$

$=$

$$39.546 = \frac{1}{H_2^{3/2}} - \frac{1}{0.3^{3/2}}.$$

$$39.546 + \frac{1}{0.3^{3/2}} = \frac{1}{H_2^{3/2}}.$$

$$H_2 = \left(\frac{1}{45.631}\right)^{2/3} = 8\ cm$$

$\boxed{H_2 = 8cm.}$

FLOW THROUGH PIPES

In a pipe flow the turbulent flow of fluids through pipes running full will be considered and the flow of fluids through pipes under pressure only.

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost.

The loss of energy is classified as

Energy Losses.

Major Energy Losses

Major energy losses in a pipe is due to friction and it is calculated by

(I) Darcy-weisbach Formula

(II) chezy's Formula

Minor Energy Losses

Minor energy losses in a pipe is due to

(I) Sudden expansion of pipe

(II) Sudden contraction of pipe

(III) Bend in pipe

(IV) pipe fittings

(V) An obstruction in pipe.

LOSS OF ENERGY OR HEAD DUE TO FRICTION

DARCY-WEISBACH FORMULA:-

$$\boxed{h_f = \frac{4fLv^2}{2gD}}$$

h_f = head loss due to friction

f = coefficient of friction which is a function of Reynold's number.

$$\boxed{f = \frac{16}{Re}}$$ $Re < 2000$ viscous flow

$$\boxed{f = \frac{0.079}{Re^{1/4}}}$$ $Re > 4000 - 10^6$

L = Length of the pipe

V = Mean velocity of flow

d = diameter of pipe

CHEZY'S FORMULA:

$$V = C\sqrt{mi}$$

$$i = \frac{h_f}{L}$$

$$\boxed{h_f = i \times L}$$

V = mean velocity of flow

A = Area of cross-section of pipe

P = Wetted perimeter of pipe

L = Length of pipe.

$m = \frac{A}{P}$ (Hydraulic mean depth)

i = Hydraulic gradient / bed slope / Loss of head per unit length of pipe.

C = Chezy's constant

MINOR ENERGY LOSSES IN A PIPE :-

The loss of energy due to change of velocity of flowing fluid in magnitude or direction is called minor loss of energy.

The minor loss of energy or head includes the following ones:

1. Loss of head due to sudden enlargement.
2. Loss of head due to sudden contraction.
3. Loss of head at entrance of pipe.
4. Loss of head at exit of pipe
5. Loss of head due to an obstruction in a pipe
6. Loss of head due to bend in a pipe
7. Loss of head in various pipe fittings.

LOSS OF HEAD AT THE ENTRANCE OF A PIPE :-

Loss of head at entrance of the pipe occurs when a liquid enters a pipe which is connected to a large tank or reservoir.

The loss is similar to the head loss due to sudden contraction.

$$\boxed{h_i = \frac{0.5V^2}{2g}}$$

V = velocity of liquid in the pipe.

LOSS OF HEAD AT THE EXIT OF PIPE :-

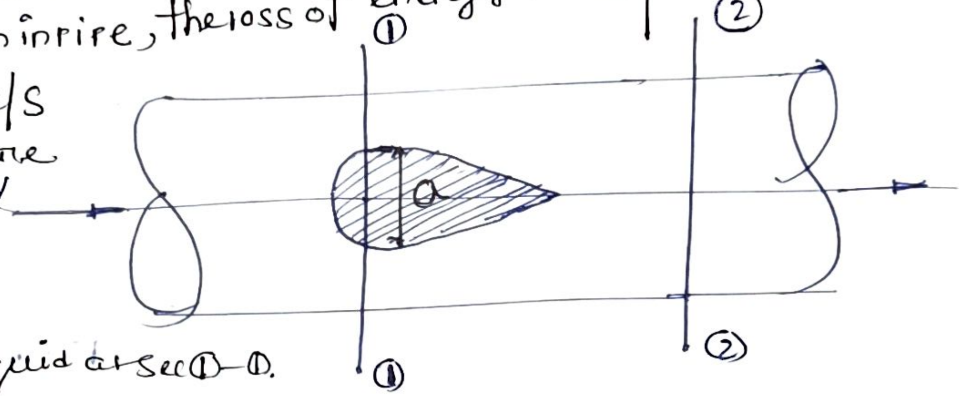
This is the loss of head or energy due to the velocity of liquid at outlet of pipe which is dissipated either in the form of a free jet or it is lost in the tank or reservoir.

$$\boxed{h_o = \frac{V^2}{2g}}$$

V = velocity at the outlet of pipe.

LOSS OF HEAD DUE TO AN OBSTRUCTION IN A PIPE :-

When there is an obstruction in pipe, the loss of energy takes place due to reduction of the area of c/s of the pipe at the place where obstruction is present.



A - a = Area of flow of liquid at sec ①-①.

h_L = Loss of head due to obstruction

$$= \frac{(V_c - V)^2}{2g}$$

A = Area of pipe

a = area of obstruction.

V = velocity of liquid in pipe.

V_c = velocity of liquid at vena contracta.

As per continuity Eqn : $A_c V_c = AV$

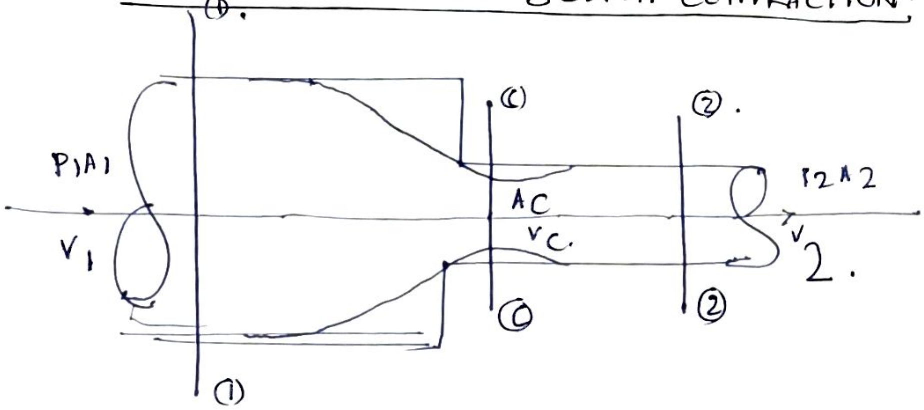
$$C_c = \frac{A_c}{A - a}$$

$$\boxed{A_c = C_c (A - a)}$$

A_c = Area of c/s at vena contracta

C_c = coefficient of contraction.

LOSS OF HEAD DUE TO SUDDEN CONTRACTION :-



$$h_c = \frac{(V_c - V_2)^2}{2g}$$

$$= \frac{V_2^2}{2g}\left(\frac{V_c}{V_2} - 1\right)^2$$

$A_cV_c = A_2V_2$ From continuity equation.

$$\frac{V_c}{V_2} = \frac{A_2}{A_c}$$

$$= \frac{1}{(A_c/A_2)} = \frac{1}{C_c} \qquad C_c = \frac{A_c}{A_2}.$$

$$\boxed{h_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2} \qquad K = \left[\frac{1}{C_c} - 1\right]^2.$$

$$= \frac{KV_2^2}{2g}$$

For. $C_c = 0.62$

$K = 0.375$

$$\boxed{h_c = 0.375\frac{V_2^2}{2g}}$$

If C_c is not given the loss of head due to contraction is given as:

$$\boxed{h_c = 0.5\frac{V_2^2}{2g}}$$

change of momentum $= \rho A_1 V_1^2 - \rho A_2 V_2^2$

$$= \rho \left(\frac{V_2}{V_1}\right) A_1 . V_1^2 - \rho A_2 V_2^2$$

$$= \rho A_1 V_1 V_2 - \rho A_2 V_2^2$$

$$= \rho \left(A_1 V_1 V_2 - V_2^2 A_2\right)$$

$$= \rho A_2 V_2^2 - \rho A_1 V_1^2$$

$$= \rho A_2 V_2^2 - \rho V_1^2 . A_2 \left(\frac{V_2}{V_1}\right).$$

$$= \rho A_2 V_2^2 - \rho V_1 V_2 A_2.$$

$$= \rho A_2 \left(V_2^2 - V_1 V_2\right)$$

Net force acting on the control volume in the direction of flow must be equal to the rate of change of momentum per second.

$$(P_1 - P_2) A_2 = \rho A_2 (V_2^2 - V_1 V_2).$$

$$\frac{P_1 - P_2}{\rho} = V_2^2 - V_1 V_2.$$

$$\boxed{\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1 V_2}{g}}$$

$$h_e = \frac{P_1 - P_2}{\rho g} + \frac{V_1^2 - V_2^2}{2g}$$

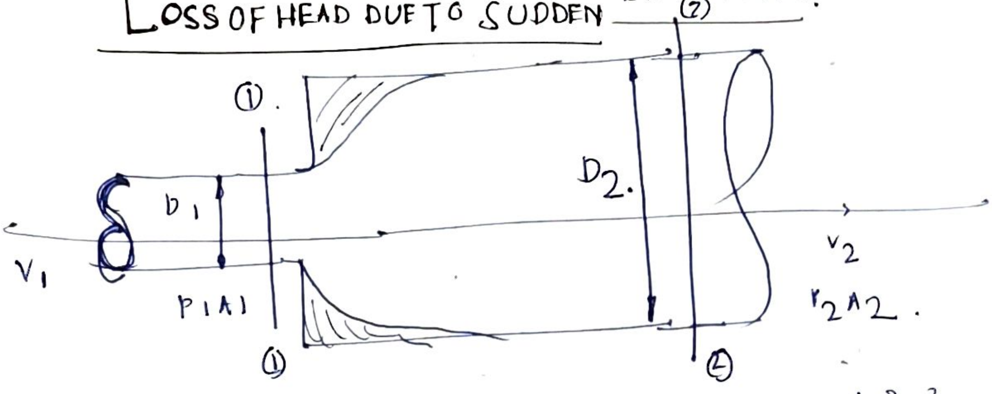
$$= \frac{V_2^2 - V_1 V_2}{g} + \frac{V_1^2 - V_2^2}{2g} = \frac{2V_2^2 - 2V_1V_2 + V_1^2 - V_2^2}{2g}$$

$$= \frac{V_1^2 - 2V_1V_2 + V_2^2}{2g}$$

$$= \frac{(V_1 - V_2)^2}{2g}$$

$$\boxed{h_e = \frac{(V_1 - V_2)^2}{2g}}.$$

LOSS OF HEAD DUE TO SUDDEN ENLARGEMENT:-



Apply Bernoulli's equation at Sections 1 & 2.

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2+h_e$$

$Z_1=Z_2$. pipe is horizontal.

h_e = Loss of head due to Sudden enlargement.

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+h_e$$

$$\boxed{h_e=\frac{P_1-P_2}{\rho g}+\frac{V_1^2-V_2^2}{2g}}$$

consider control volume of liq b/w sec 1 & 2.

Due to sudden change of diameter of pipe from D_1 to D_2, the liquid flowing from the smaller pipe is not able to follow the abrupt change of boundary. The flow separates from boundary & turbulent eddies are formed.

$$F_x=P_1A_1-P_2A_2+P'(A_2-A_1)$$
$$=P_1A_1-P_2A_2+P_1(A_2-A_1)$$
$$=\cancel{P_1A_1}-P_2A_2+P_1A_2-\cancel{P_1A_1}$$

$$\boxed{F_x=(P_1-P_2)A_2}$$

Momentum of liquid at sec 1 = Mass x velocity
$$=\rho A_1V_1\times V_1=\rho A_1V_1^2.$$

Momentum of liquid at sec 2 $=\rho A_2V_2\times V_2=\rho A_2V_2^2.$

Change of momentum $=\rho A_2V_2^2-\rho A_1V_1^2$

From continuity equation: $A_1V_1=A_2V_2.$

$$\boxed{A_1=\left(\frac{V_2}{V_1}\right)\times A_2}$$

$$V_c \times C_c (A-a) = AV.$$

$$\boxed{V_c = \frac{AV}{(A-a)C_c}.}$$

$$h_L = \frac{(V_c - V)^2}{2g} = \frac{\left[\frac{AV}{(A-a)C_c}\right]^2 - V^2}{2g.}$$

$$= \frac{V^2}{2g}\left[\frac{A}{C_c(A-a)} - 1\right]^2$$

$$\boxed{h_L = \frac{V^2}{2g}\left[\frac{A}{C_c(A-a)} - 1\right]^2}$$

HYDRAULLIC GRADIENT LINE AND TOTAL ENERGY LINE :-

(I). Hydraullic Gradient Line is the line which gives the sum of pressure head and datum head of a flowing fluid in a pipe w.r.t some reference line or it is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head of a flowing fluid in a pipe from the centre of the pipe.

$$HGL = \frac{P}{\rho g} + Z$$

$= $ pressure head + datum head.

TOTAL ENERGY LINE: (TEL):-

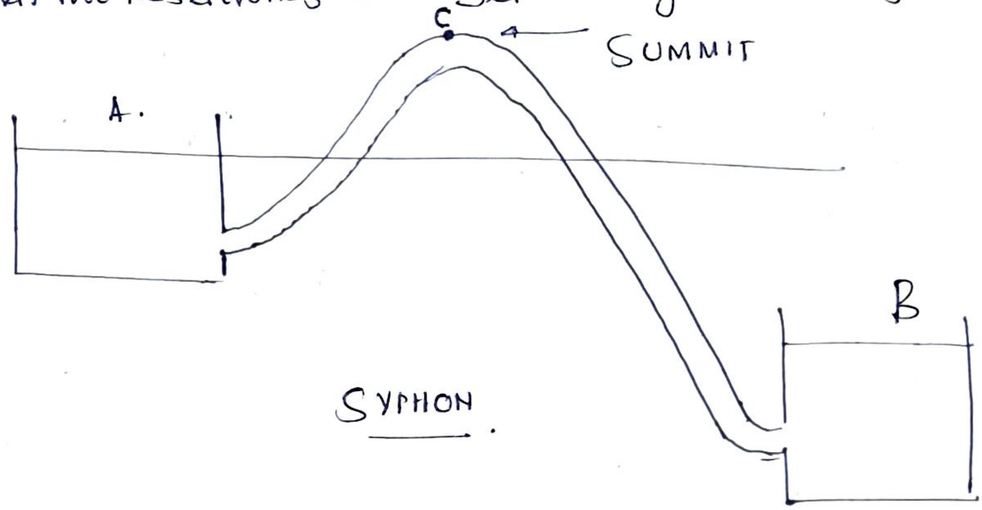
(I). Total Energy Line is the line which gives the sum of pressure head, datum head and kinematic head of a flowing fluid in a pipe with respect to some reference line.

(II). It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe.

$$TEL = \frac{P}{\rho g} + \frac{V^2}{2g} + Z$$

Total Energy Line = pressure head + velocity head + datum head

Syphon is a long bent pipe which is used to transfer liquid from a reservoir at higher elevation to a reservoir at lower elevation when two reservoirs are separated by hill or high level ground.

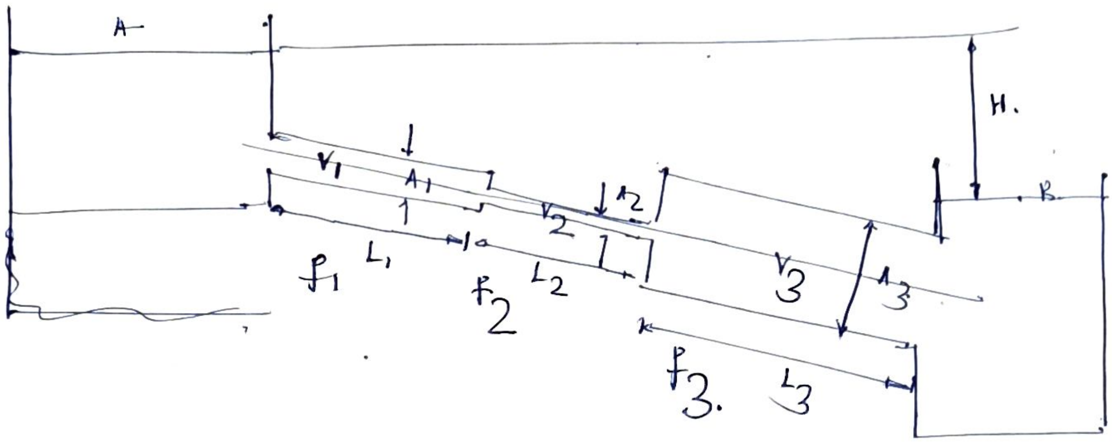


SYPHON.

The point 'c' which is the highest point of syphon is called summit.
The pressure at c will be less than atmospheric pressure.

1. Syphon is used to carry water from one reservoir to another reservoir separated by a hill or ridge.
2. To take out the liquid from a tank which is not having any outlet.
3. To empty a channel ~~which is~~ not provided with any outlet sluice.

Pipes in series or compound pipes are defined as the pipes of different lengths and different diameters connected end to end in series to form a pipe.



$Q = A_1V_1 = A_2V_2 = A_3V_3$

The difference in liquid surface levels in equal to the sum of the total head loss in the pipes.

$$H = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{2gd_1} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{2gd_2} + \frac{(V_2-V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{2gd_3} + \frac{V_3^2}{2g}$$

$$= \frac{4f_1L_1V_1^2}{2gd_1} + \frac{4f_2L_2V_2^2}{2gd_2} + \frac{4f_3L_3V_3^2}{2gd_3} \quad \text{(Neglecting minor losses)}$$

$$H = \frac{4f}{2g}\left[\frac{L_1V_1^2}{d_1} + \frac{L_2V_2^2}{d_2} + \frac{L_3V_3^2}{d_3}\right]$$

$$\boxed{H = \frac{2f}{g}\left[\frac{L_1V_1^2}{d_1} + \frac{L_2V_2^2}{d_2} + \frac{L_3V_3^2}{d_3}\right]}$$

This is defined as the pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe consisting of several pipes.

2. Find the diameter of a pipe of length 2000m when the rate of flow of water through the pipe is 200 Lits/sec and head lost due to friction is 4m. C = 50

$L = 2000m.$

$Q = 200 \text{ Litres/sec}$

$= 0.2 \, m^3/sec.$

$h_L = 4m.$

$C = 50.$

$$i = \frac{h_L}{L} = \frac{4}{2000} = \frac{1}{\left(\frac{2000}{4}\right)} = \frac{1}{500}$$

$$\boxed{i = \frac{1}{500}}$$

$$Q = AC\sqrt{mi}$$

$$= C \times \frac{\pi}{4} \times D^2 \times \sqrt{\left(\frac{D}{4}\right) \times i}$$

$$0.3 = 50 \times \frac{\pi}{4} \times \sqrt{\frac{1}{500}} \times \frac{1}{2} \, D^2 \cdot D^{1/2}.$$

$$\frac{0.3}{50 \times \frac{\pi}{4} \times \sqrt{\frac{1}{500}} \times \frac{1}{2}} = D^{5/2}.$$

$$D = \left(\frac{0.3}{50 \times \frac{\pi}{4} \times \sqrt{\frac{1}{500}} \times \frac{1}{2}}\right)^{2/5}$$

$= 0.15 \, m.$

$$\boxed{D = 650mm.}$$

1. Find the head lost due to friction in a pipe of diameter 300mm and length 50m. through which water is flowing at a velocity of 3 m/s.

(I) Using Darcy's Formula.

(II) using Chezy's Formula $C = 60$. $\nu = 0.01$ stoke $= 0.01 \times 10^{-4}\ m^2/sec$.

Darcy-weisbach Formula:

(I). $D = 300mm = 0.3m$

$V = 3\ m/s.$

$L = 50m.$

$C = 60.$

$h_L = \frac{4fLV^2}{2gD}$

$= \frac{4 \times 0.00256 \times 50 \times 3^2}{2 \times 9.81 \times 0.3}$

$= 0.7828m.$

$\boxed{h_L = 0.7828m.}$

$Re = \frac{VD}{\nu}$

$= \frac{3 \times 0.3}{1 \times 10^{-6}}$

$= 900000 = 9 \times 10^5 > 4000.$

(Turbulent Flow)

$f = \frac{0.079}{Re^{1/4}}$

$= \frac{0.079}{(9 \times 10^5)^{0.25}} = 0.00256$

$\boxed{f = 0.00256}$

Chezy's Formula:

(II). $V = C\sqrt{mi}$

$i = \frac{h_L}{L}$ (Hydraulic gradient/Bedslope)

C = Chezy's constant $= 60$

m = Hydraulic mean depth.

i = channel slope/Bedslope.

$m = \frac{A}{P} = \frac{\frac{\pi}{4} \times D^2}{\pi D}$

$= \frac{\frac{\pi}{4} \times 0.3^2}{\pi \times 0.3} = 0.075m.$

$\boxed{m = 0.075m}$

$h_L = i \times L$

$= \frac{1}{30} \times 50$

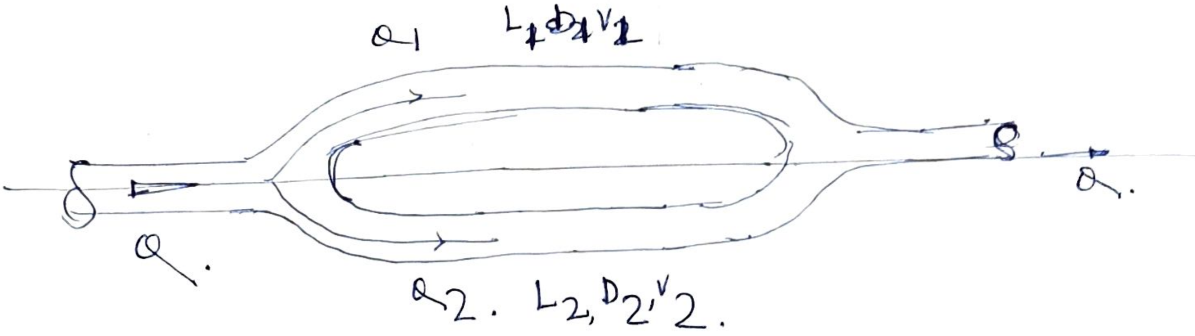
$= 1.66m.$

$\boxed{h_L = 1.66m.}$

$3 = 60\sqrt{0.075 \times i}$

$i = \left(\frac{3}{60\sqrt{0.075}}\right)^2 = (0.1825)^2 = \left(\frac{1825}{10000}\right)^2 = \frac{1}{\left(\frac{10000}{1825}\right)^2} = \frac{1}{30}$

$\boxed{i = \frac{1}{30}}$



$Q = Q_1 + Q_2$

Loss of head at branch pipe 1 = Loss of head at branch pipe 2.

$$\frac{4 f_1 L_1 V_1^2}{2 g D_1} = \frac{4 f_2 L_2 V_2^2}{2 g D_2}.$$

$f_1 = f_2$

$$\frac{4 f L_1 V_1^2}{2 g D_1} = \frac{4 f L_2 V_2^2}{2 g D_2}$$

The uniform diameter of equivalent pipe is called equivalent size of pipe.

The length of equivalent pipe is equal to the sum of lengths of the compound pipe consisting of different pipes.

$$H = \frac{4f_1 L_1 V_1^2}{2gD_1} + \frac{4f_2 L_2 V_2^2}{2gD_2} + \frac{4f_3 L_3 V_3^2}{2gD_3} \qquad f_1 = f_2 = f_3 = f$$

$$Q = A_1V_1 = A_2V_2 = A_3V_3 = \left(\frac{\pi}{4}D_1^2\right)\times V_1 = \left(\frac{\pi}{4}\times D_2^2\right)\times V_2 = \left(\frac{\pi}{4}\times D_3^2\right)\times V_3$$

$$V_1 = \frac{4Q}{\pi D_1^2} \qquad V_2 = \frac{4Q}{\pi D_2^2} \qquad V_3 = \frac{4Q}{\pi D_3^2}$$

$$H = \frac{4f_1L_1\left(\frac{4Q}{\pi D_1^2}\right)^2}{2gD_1} + \frac{4f_2L_2\left(\frac{4Q}{\pi D_2^2}\right)^2}{2gD_2} + \frac{4f_3L_3\left(\frac{4Q}{\pi D_3^2}\right)^2}{2gD_3}$$

$$= \frac{4f\times 16Q^2}{2g\times\pi^2}\left(\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}\right)$$

$$H = \frac{4fLV^2}{2gD} = \frac{4fL\left(\frac{Q}{A}\right)^2}{2gD} = \frac{4fL\times 16Q^2}{2g\times\pi^2\times D^5}$$

$$= \frac{4f\times 16Q^2}{2g\pi^2}\left(\frac{L}{D^5}\right).$$

$$\boxed{\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}}$$

3. A crude oil of kinematic viscosity 0.4 stoke is flowing through a pipe of diameter 300mm at the rate of 300 Litres/sec. Find the head lost due to friction for a length of 50m of the pipe.

$\nu = 0.4\text{ stoke}$

$= 0.4\times10^{-4}\ m^2/sec$

$Q = 300\ \text{Litres/sec.} = 0.3\ \frac{m^3}{sec}$

$D = 300mm = 0.3m$

$L = 50m.$

$V = \frac{Q}{A} = \frac{0.3}{\frac{\pi}{4}\times0.3^2} = 4.244\ m/s.$

$\boxed{V = 4.244\ m/s.}$

$R_e = \frac{VD}{\nu}$

$= \frac{4.244\times0.3}{0.4\times10^{-4}}$

$= 31830 > 4000.$ The flow is turbulent.

$\boxed{R_e = 31830}$

$f = \frac{0.079}{(R_e)^{1/4}} = \frac{0.079}{(31830)^{0.25}} = 0.00591$

$\boxed{f = 0.00591}$

$h_L = \frac{4fLV^2}{2gD} = \frac{4\times0.00591\times50\times4.244^2}{2\times9.81\times0.3}$

$= 3.616m$

$\boxed{h_L = 3.616m}$

$\sqrt{2g\,H}$

9. An oil of specific gravity 0.7 is flowing through a pipe of diameter 300 mm at the rate of 500 litres/sec. Find the head lost due to friction and power required to maintain the flow for a length of 1000 m. $\nu = 0.29$ stokes.

$S = 0.7$

$\rho = 700\,kg/m^3$

$D = 300\,mm$
$= 0.3\,m.$

$Q = 500$ litres/sec
$= 0.5\,m^3/sec.$

$L = 1000\,m.$

$\nu = 0.29$ stokes
$= 0.29 \times 10^{-4}\,m^2/sec.$

$V = \dfrac{Q}{A}$

$= \dfrac{0.5}{\frac{\pi}{4} \times 0.3^2}$

$= 7.0735\,m/s.$

$\boxed{V = 7.0735\,m/s.}$

$R_e = \dfrac{VD}{\nu} = \dfrac{7.0735 \times 0.3}{0.29 \times 10^{-4}}$

$= 73174 > 4000.$

$\boxed{R_e = 73174.}$

$f = \dfrac{0.0079}{R_e^{1/4}} = \dfrac{0.079}{(73174)^{0.25}} = 0.0048$

$\boxed{f = 0.0048.}$

$h_L = \dfrac{4fLV^2}{2gD} = \dfrac{4 \times 0.0048 \times 1000 \times 7.0735^2}{2 \times 9.81 \times 0.3}$

$= 163.21\,m$

$\boxed{h_L = 163.21\,m}$

Power Required:

$P = \gamma_w Q H.$

$= \rho_w g Q H.$

$= 700 \times 9.81 \times 0.5 \times 163.21$

$= 560.38\,kW.$

$\boxed{P = 560.38\,kW.}$

5. Calculate the discharge through a pipe of diameter 200mm when the difference of pressure head b/w two ends of a pipe 500m apart is 1m of water. Take $f = 0.009$.

$D = 200mm = 0.2m$. $f = 0.009$.

$L = 500m$.

$h_L = 1m$.

$$V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}D^2} = \frac{4Q}{\pi D^2}$$

$$h_L = \frac{4fLv^2}{2gD}.$$

$$1 = \frac{4 \times 0.009 \times 500 \times \left(\frac{4Q}{\pi D^2}\right)^2}{2 \times 9.81 \times D}.$$

$$\sqrt{\frac{1 \times 2 \times 9.81 \times \pi^2 \times 0.2^5}{4 \times 0.009 \times 500 \times 16}} = Q.$$

$Q = 0.0293 \ m^3/sec.$

$$\boxed{Q = 0.0293 \ m^3/sec.}$$

6. Water is flowing through a pipe of diameter 200mm with a velocity of 3 m/s. Find the head lost due to friction for a length of 5m if the coefficient of friction is given by $\boxed{f = 0.02 + \frac{0.09}{Re^{0.3}}}$. Where Re = Reynold's Number. The kinematic viscosity of water = 0.01 stoke.

$D = 200mm = 0.2m$.

$V = 3 \ m/s$.

$L = 5m$

$\nu = 0.01 \times 10^{-4} \ m^2/sec.$

$$Re = \frac{VD}{\nu} = \frac{3 \times 0.2}{0.01 \times 10^{-4}} = 600 \times 10^3 > 4000$$

$$\boxed{Re = 600 \times 10^3}$$

$$f = 0.02 + \frac{0.09}{R_e^{0.3}}$$

$$= 0.02 + \frac{0.09}{(600\times10^3)^{0.3}} = 0.02166$$

$$\boxed{f = 0.02166}$$

$$h_L = \frac{4fLV^2}{2gD} = \frac{4\times0.02166\times5\times3^2}{2\times9.81\times0.2}$$

$$= 0.9908\ m.$$

$$\boxed{h_L = 0.9908m.}$$

7. An oil of specific gravity 0.9 and viscosity 0.06 poise is flowing through a pipe of diameter 200mm at the rate of 60 Ltrs/sec. Find the head lost due to friction for a 500m length of pipe. Find the power required to maintain this flow

$S = 0.9.$

$\rho = 0.9\times1000 = 900\ kg/m^3.$

$\mu = 0.06$ poise

$= 0.06\times0.1\ \frac{N\ sec}{m^2}$

$= 0.006\ \frac{N\text{-}sec}{m^2}$

$$\boxed{\mu = 0.006\ \frac{N\text{-}sec}{m^2}}$$

$D = 200mm = 0.2m$

$Q = 60\ Ltrs/sec = 0.06\ m^3/sec.$

$L = 500m.$

$$V = \frac{0.06}{\frac{\pi}{4}\times0.2^2} = 1.90\ m/sec.$$

$$R_e = \frac{\rho VD}{\mu} = \frac{900\times1.90\times0.2}{0.006}$$

$\simeq 57000 > 4000.$

(The flow is turbulent)

$$\boxed{R_e = 57000}$$

$$f = \frac{0.079}{R_e^{0.14}} = \frac{0.079}{(57000)^{0.25}} = 0.00511$$

$$\boxed{f = 0.00511}$$

$$h_L = \frac{4\times0.00511\times500\times1.90^2}{2\times9.81\times0.2}$$

$= 9.4m.$ $\boxed{h_L = 9.4m}$

$P = 900\times9.81\times0.06\times9.4 = 5\ Kw.$

(iii). power lost due to sudden enlargement:

$P = \gamma_w Q H_L = \rho_w g Q (H_L)$.

$= 1000 \times 9.81 \times 0.25 \times 1.817$

$= 4.456$ kW.

$$\boxed{P = 4.456 \text{ kW.}}$$

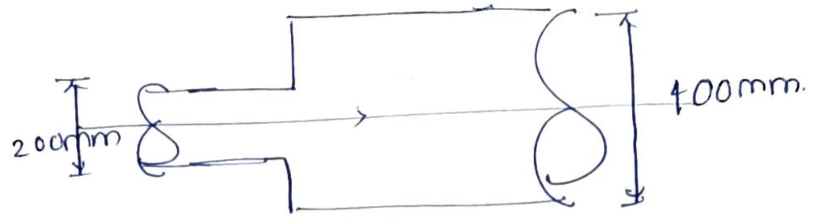
10. The rate of flow of water through a horizontal pipe is $0.25\ m^3/sec$. The diameter of the pipe which is 200mm is suddenly enlarged to 400mm. The pressure intensity in the smaller pipe is $11.772\ N/cm^2$.

(I) Determine loss of head due to sudden enlargement.

(II) pressure intensity in the larger pipe.

(III) power lost due to enlargement.

$q = 0.25 m^3/sec.$



$D_1 = 200mm$

$= 0.2m$

$D_2 = 400mm$

$= 0.4m.$

$A_1 = \frac{\pi}{4} \times 0.2^2 = 0.0314 m^2$

$A_2 = \frac{\pi}{4} \times 0.4^2 = 0.1256 m^2$

$V_1 = 7.961 m/s.$

$V_2 = 1.990 m/s.$

$P_1 = 11.772\ N/cm^2$

$= 11.772 \times 10^4\ N/m^2$

(I) $h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.961 - 1.99)^2}{2 \times 9.81}$

$= 1.817m.$

$\boxed{h_e = 1.817m}$

(II). $\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_e.$

$\left[\frac{11.772 \times 10^4}{1000 \times 9.81} + \frac{7.961^2}{2 \times 9.81} - \frac{1.99^2}{2 \times 9.81} - 1.817\right] 1000 \times 9.81 = P_2$

Rise of hydraulic gradient $\left(\frac{P_2}{\rho g}+Z_2\right)-\left(\frac{P_1}{\rho g}+Z_1\right)=10\,mm=\frac{10}{1000}=\frac{1}{100}$

Apply Bernoulli's Theorem to both sections:

$$\left(\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1\right)=\left(\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2\right)+h_e$$

$$h_e=\frac{(V_1-V_2)^2}{2g}$$ ← head loss due to enlargement.

$A_1V_1=A_2V_2$ apply continuity eq^n.

$$V_1=\left(\frac{A_2}{A_1}\right)V_2$$

$$=\frac{\left(\frac{\pi}{4}\times 0.48^2\right)}{\left(\frac{\pi}{4}\times 0.24^2\right)}\times V_2=4V_2$$

$$\boxed{V_1=4V_2}$$

$$h_e=\frac{(4V_2-V_2)^2}{2g}=\frac{(3V_2)^2}{2g}=\frac{4.5V_2^2}{g}$$

$$\frac{P_1}{\rho g}+\frac{(4V_2)^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2+\frac{9V_2^2}{2g}$$

$$\frac{P_1}{\rho g}+\frac{16V_2^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2+\frac{9V_2^2}{2g}$$

$$\boxed{\left(\frac{P_2}{\rho g}+Z_2\right)-\left(\frac{P_1}{\rho g}+Z_1\right)=\frac{6V_2^2}{2g}}$$

$$\frac{6V_2^2}{2g}=\frac{1}{100}$$

$$V_2=\sqrt{\frac{9.81\times2}{600}}=0.18\,m/s$$

$$\boxed{V_2=0.18\,m/sec}$$

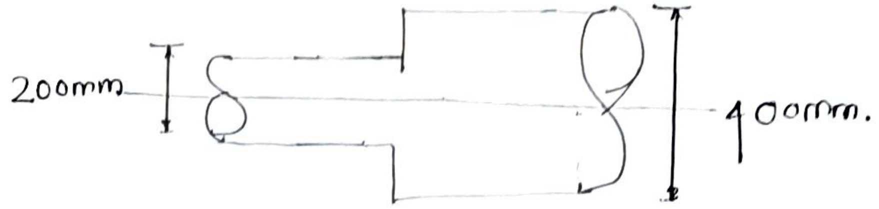
$$Q=A_2V_2$$

$$=\left(\frac{\pi}{4}\times0.48^2\right)\times0.18$$

$$=0.0325\,m^3/sec$$

$$\boxed{Q=0.0325\,m^3/sec}$$

8. Find the loss of head when a pipe of diameter 200mm is suddenly enlarged to a diameter of 400mm. The rate of flow of water through the pipe is 250 Litres/sec.



$D_1 = 200mm$
$= 0.2m$

$D_2 = 400mm$
$= 0.4m$

$Q = 250 \text{ Litres/sec} = 0.25 m^3/sec.$

$$V_1 = \frac{Q}{A_1} = \frac{0.25}{\frac{\pi}{4} \times 0.2^2} = \frac{0.25}{0.0314} = 7.961 m/s.$$

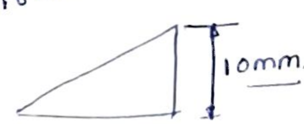
$$V_2 = \frac{Q}{A_2} = \frac{0.25}{\frac{\pi}{4} \times 0.4^2} = \frac{0.25}{0.1256} = 1.990 m/s.$$

Loss of head due to sudden enlargement: $h_e = \frac{(V_1 - V_2)^2}{2g}$

$$= \frac{(7.961 - 1.990)^2}{2 \times 9.81} = 1.817m.$$

$$\boxed{h_e = 1.817m.}$$

9. At a sudden enlargement of a water main from 240mm to 480mm diameter, the hydraulic gradient rises by 10mm. Estimate the rate of flow.



$D_1 = 240mm$
$= 0.24m.$

$D_2 = 480mm$
$= 0.48m.$

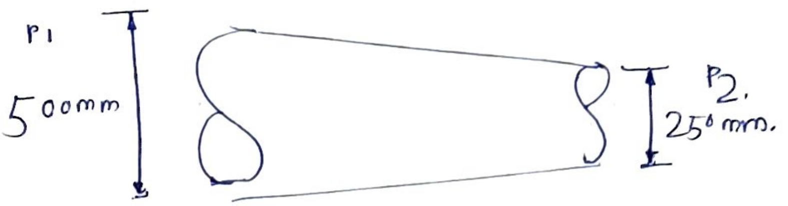
$A_1 = \frac{\pi}{4} \times 0.24^2 = 0.0452 m^2$

$A_2 = \frac{\pi}{4} \times 0.48^2 = 0.1809 m^2.$

11. A horizontal pipe of diameter 500mm is suddenly contracted to diameter of 250mm. The pressure intensities in the large and smaller pipe is given 13.734 N/cm^2 and 11.772 N/cm^2 respectively. Find the loss of head due to contraction if $C_c = 0.62$. Also determine rate of flow of water.

$P_1 = 13.734 \times 10^4 \, N/m^2$

$P_2 = 11.722 \times 10^4 \, N/m^2$



$D_1 = 500mm$
$= 0.5m$

$D_2 = 250mm$
$= 0.25m.$

$A_1 = \frac{\pi}{4} \times 0.5^2 = 0.1963 m^2$

$A_2 = \frac{\pi}{4} \times 0.25^2 = 0.0490 m^2$

$C_c = 0.62$

$$h_c = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$$

$$= \frac{V_2^2}{2g}\left(\frac{1}{0.62} - 1\right)^2 = 0.375 \frac{V_2^2}{2g}$$

$$\boxed{h_c = 0.375 \frac{V_2^2}{2g}}$$

From continuity Equation :- $A_1 V_1 = A_2 V_2$.

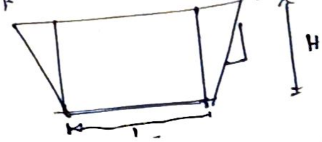
$$V_2 = \frac{\left(\frac{\pi}{4} \times 0.5^2\right) \times V_1}{\left(\frac{\pi}{4} \times 0.25^2\right)}$$

$$= 4V_1$$

$$\boxed{V_2 = 4V_1}$$

Apply Bernoulli's Equation :-

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_c$$



$\sqrt{2g}\ H^{5/2}$.

$$\frac{13.734\times10^4}{1000\times9.81} + \frac{(V_2/4)^2}{2\times9.81} = \frac{11.772\times10^4}{1000\times9.81} + \frac{V_2^2}{2\times9.81} + \frac{0.375V_2^2}{2\times9.81}$$

$$\frac{-0.0625V^2 + V^2 + 0.375V^2}{2\times9.81} = \frac{(13.7340-11.722)\times10^4}{1000\times9.81}$$

$$1.3125V^2 = 2.012\times10\times2$$

$$V = \sqrt{\frac{2.012\times20}{1.3125}}$$

$$= 5.537 \text{ m/s.}$$

$$\boxed{V = 5.537 \text{ m/s.}}$$

(i) Loss of head due to contraction: $h_c = \frac{0.375V_2^2}{2g}$

$$= \frac{0.375\times5.537^2}{2\times9.81}$$

$$= 0.585 \text{ m.}$$

$$\boxed{h_c = 0.585\text{m.}}$$

(ii). Rate of flow of water: $Q = A_2V_2$

$$= 0.0490\times5.537 = 0.271 \text{ m}^3/\text{sec.}$$

$$\boxed{Q = 0.271 \text{ m}^3/\text{sec}}$$

12. A 150mm diameter pipe reduces in diameter abruptly to 100mm diameter. If the pipe carries water at 30 Litres/sec, calculate the pressure loss across the contraction. Take the coefficient of contraction as 0.6.

$D_1 = 150mm = 0.15m \quad A_1 = \frac{\pi}{4} \times D_1^2$

$D_2 = 100mm. = 0.1m. \quad = \frac{\pi}{4} \times 0.15^2 = 0.0176m^2$

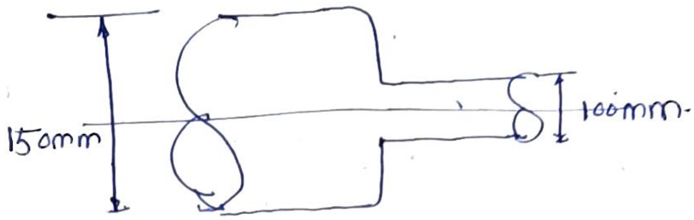
$Q = 30$ Litres/sec $\quad A_2 = \frac{\pi}{4} \times D_2^2 = \frac{\pi}{4} \times 0.1^2$

$= 0.03 m^3/sec. \quad = 0.00785m^2$

$C_c = 0.6$

$V_1 = \frac{0.03}{0.0176}$

$= 1.70 m/s.$



$V_2 = \frac{0.03}{0.00785} = 3.82 m/s.$

Apply Bernoulli's Theorem before and after contraction:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_c.$$

$Z_1 = Z_2$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + h_c.$$

$$h_c = \frac{V_2^2}{2g}\left(\frac{1}{C_c^2} - 1\right).$$

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g} + h_c$$

$$= \frac{3.82^2}{2 \times 9.81} \times \left(\frac{1}{0.6^2} - 1\right)$$

$$= 0.4558m.$$

$$\boxed{h_c = 0.4558m.}$$

$$P_1 - P_2 = \left(\frac{3.82^2 - 1.70^2}{2 \times 9.81} + 0.4558\right) \times 1000 \times 9.81$$

$$= 10.32 \times 10^3 N/m^2$$

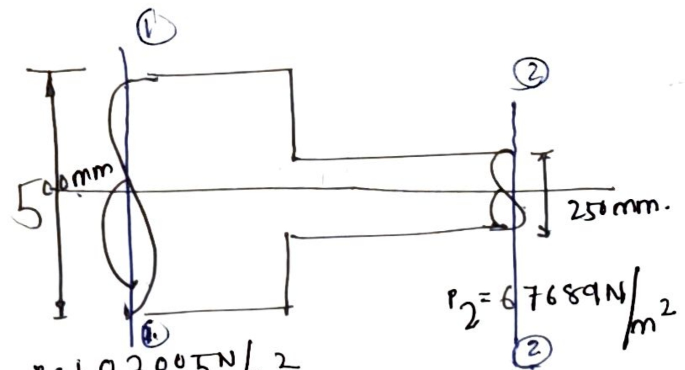
$$= 1.032 \times 10^4 N/m^2$$

$$P_1 - P_2 = 1.032 N/cm^2$$

$$\boxed{P_1 - P_2 = 1.032 N/cm^2}$$

13. When a sudden contraction is introduced in a horizontal pipeline from 500mm to 250mm, the pressure changes from 103005 N/m^2 to 67689 N/m^2.
Calculate the rate of flow. Assume coefficient of contraction of jet to be 0.65.

Following this if there is a sudden enlargement from 250mm to 500mm and if the pressure at the 250mm section is 67689 N/m^2. What is the pressure at the 500mm enlarged section?



$D_1 = 500mm = 0.5m$

$A_1 = \frac{\pi}{4} \times 0.5^2 = 0.1963 m^2$

$D_2 = 250mm = 0.25m$

$A_2 = \frac{\pi}{4} \times 0.25^2 = 0.0490m^2$

$C_c = 0.65.$

$$h_c = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$$

$$= \frac{V_2^2}{2g}\left(\frac{1}{0.65} - 1\right)^2$$

$$= 0.289 \frac{V_2^2}{2g}$$

$$\boxed{h_c = 0.289 \frac{V_2^2}{2g} = \frac{0.3 V_2^2}{2g}}$$

Apply continuity eq^n:

$A_1V_1 = A_2V_2$

$V_2 = \frac{A_1V_1}{A_2} = 4V_1.$

$$\boxed{V_2 = 4V_1}$$

Apply Bernoulli's Theorem at sec: 1 & 2.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_c.$$

$P_1 = 103005 N/m^2$

$P_2 = 67689 N/m^2$

$$\frac{103005}{1000 \times 9.81} + \frac{(V_2/4)^2}{2 \times 9.81} = \frac{67689}{1000 \times 9.81} + \frac{V_2^2}{2g} + \frac{0.3V_2^2}{2g}$$

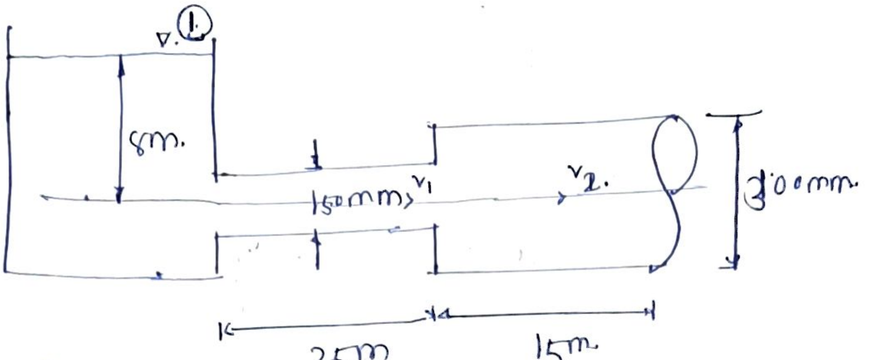
$$\frac{103005 - 67689}{1000 \times 9.81} = \frac{V_2^2 + 0.3V_2^2 - 0.0625V_2^2}{2g}$$

$$\frac{35316}{1000 \times 9.81} \times 2 \times 9.81 = 1.2375 V_2^2$$

$$V_2 = \sqrt{\frac{70.632}{1.2375}} = 7.55 m/s$$

$$\boxed{V_2 = 7.55 m/s}$$

A horizontal pipe line 40m long is connected to a water tank at one end and discharges freely into atmosphere at other end. For the first 25m of its length from the tank, the pipe is 150mm diameter and its diameter suddenly enlarged to 300mm. The height of water level in the tank is 8m above the centre of the pipe. Consider all losses of head which occur, determine the rate of flow. Take $f = 0.01$ for all sections of pipe.



$f = 0.01$.

$L_1 = 25m$ $\quad D_1 = 150mm = 0.15m$

$L_2 = 15m$ $\quad D_2 = 300mm = 0.3m$.

Losses:

$$h_i = \frac{0.5V_1^2}{2g} = 0.025V_1^2$$

$$h_{f1} = \frac{4fL_1V_1^2}{2gD_1} = \frac{4\times0.01\times25\times V_1^2}{2\times9.81\times0.15} = 0.34V_1^2$$

$$h_{f2} = \frac{4fL_2V_2^2}{2gD_2} = \frac{4\times0.01\times15\times V_2^2}{2\times9.81\times0.3} = 0.1019V_2^2$$

$$h_e = \frac{(V_1-V_2)^2}{2g} = \frac{(V_1-V_2)^2}{2g}.$$

Apply Bernoulli's Theorem b/w free surface of water and outlet of pipe:

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1 = \frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2+h_L$$

Apply continuity Eqn:-

$A_1V_1 = A_2V_2$

$V_1 = 4V_2$.

$$0+0+8 = 0+\frac{V_2^2}{2g}+0+0.025V_1^2+0.34V_1^2+0.1019V_2^2+\frac{(V_1-V_2)^2}{2g}$$

$$8 = \frac{V_2^2}{2g}+0.025(4V_2)^2+0.34V_2^2 \times 16+0.1019V_2^2+\frac{(3V_2)^2}{2g}$$

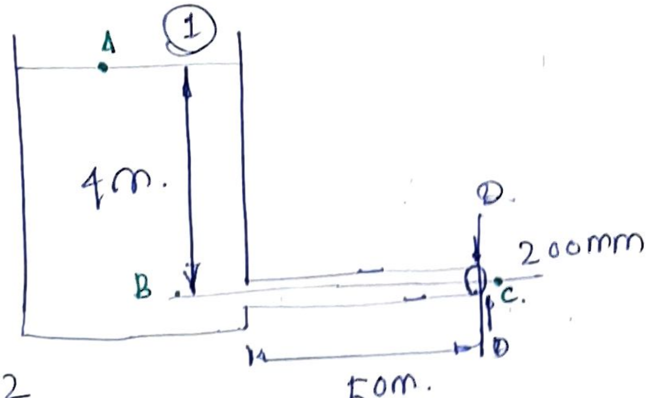
Determine the rate of flow of water through a pipe of diameter 200mm and length 50m when one end of the pipe is connected to a tank and other end of pipe is connected to atmosphere. The pipe is horizontal and the height of water in the tank is 4m above the center of the pipe.

Consider all minor losses and take $f = 0.009$ and $h_f = \frac{4fLv^2}{2gD}$.

$D = 200mm = 0.2m$

$L = 50m$

$f = 0.009$.



$$h_I = \frac{0.5V_2^2}{2g} = 0.025V^2$$

$$h_f = \frac{4fLV^2}{2gD} = \frac{4\times0.009\times50V^2}{2\times9.81\times0.2}$$

$$= \cancel{0.45V^2}\ 0.458V^2$$

Apply Bernoulli's Theorem b/w 1 & 2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_I + h_f$$

$$0 + 0 + 4 = 0 + \frac{V^2}{2g} + 0 + 0.458V^2 + 0.025V^2$$

$$V = \sqrt{\frac{4}{(0.05 + 0.458 + 0.025)}} = 2.739 m/s.$$

$$\boxed{V = 2.739 m/s.}$$

$$Q = AV. = \left(\frac{\pi}{4}\times0.2^2\right)\times2.739$$

$$= 0.086 m^3/sec$$

$$\boxed{Q = 0.086 m^3/sec.}$$

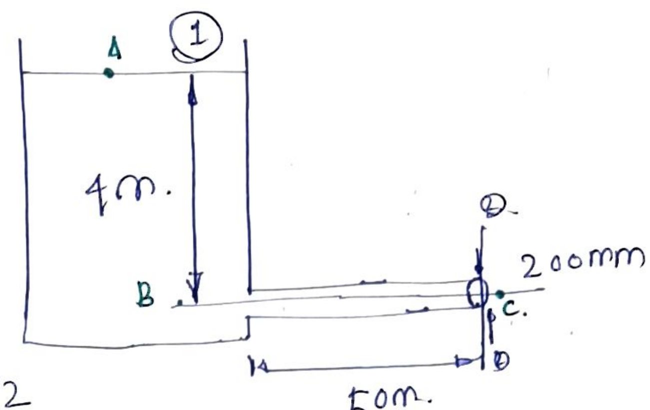
Determine the rate of flow of water through a pipe of diameter 200mm and length 50m when one end of the pipe is connected to a tank and other end of pipe is connected to atmosphere. The pipe is horizontal and the height of water in the tank is 4 m above the center of the pipe.

Consider all minor losses and take $f = 0.009$ and $h_f = \frac{4fLv^2}{2gD}$.

$D = 200mm = 0.2m$

$L = 50m$

$f = 0.009$.



$$h_I = \frac{0.5V^2}{2g} = 0.025V^2$$

$$h_f = \frac{4fLV^2}{2gD} = \frac{4 \times 0.009 \times 50 V^2}{2 \times 9.81 \times 0.2}$$

$$= \cancel{0.45V^2}\ 0.458V^2$$

Apply Bernoulli's Theorem b/w 1 & 2

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + Z_2 + h_I + h_f$$

$$0 + 0 + 4 = 0 + \frac{V^2}{2g} + 0 + 0.458V^2 + 0.025V^2$$

$$V = \sqrt{\frac{4}{(0.05 + 0.458 + 0.025)}} = 2.739 m/s.$$

$$\boxed{V = 2.739\ m/s.}$$

$$Q = AV. = \left(\frac{\pi}{4} \times 0.2^2\right) \times 2.739$$

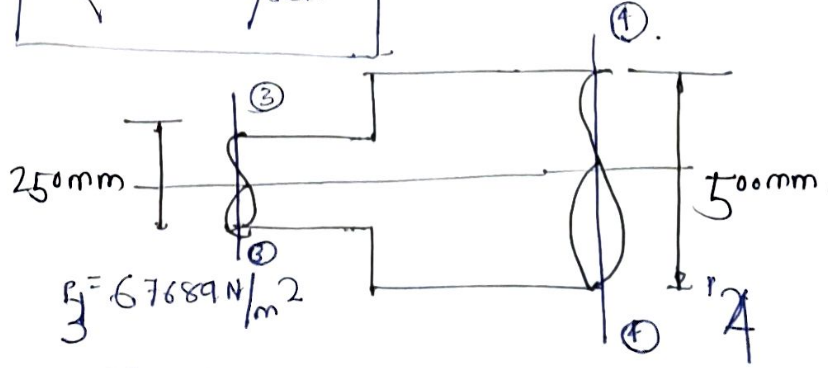
$$= 0.086\ m^3/sec$$

$$\boxed{Q = 0.086\ m^3/sec}$$

$Q = A_2 V_2$

$= 0.049 \times 7.55 = 0.37 \, m^3/sec.$

$$\boxed{Q = 0.37 \, m^3/sec.}$$



Apply Bernoulli's eqn b/w ③ & ④.

$$\frac{P_3}{\rho g} + \frac{V_2^2}{2g} + Z_3 = \frac{P_4}{\rho g} + \frac{V_4^2}{2g} + Z_4 + h_e.$$

$Z_3 = Z_4.$

$$\frac{67689}{1000 \times 9.81} + \frac{7.55^2}{2 \times 9.81} = \frac{P_4}{\rho g} + \frac{1.8875^2}{2 \times 9.81} + 1.634 \, m.$$

$$\left(\frac{67689}{10^3 \times 9.81} + \frac{7.55^2 - 1.8875^2}{2 \times 9.81} - 1.634\right) \times \rho g = P_4$$

$P_4 = 78.4 \times 10^3 \, N/m^2$

$$\boxed{\begin{aligned} P_4 &= 7.84 \times 10^4 \, N/m^2 \\ &= 7.84 \, N/cm^2 \end{aligned}}$$

$$h_e = \frac{(V_3 - V_4)^2}{2g}$$

$$= \frac{(7.55 - 1.8875)^2}{2 \times 9.81}$$

$= 1.634 \, m$

$$\boxed{h_e = 1.634 \, m}$$

$V_3 = V_2 = 7.55 \, m/s.$

$V_4 = \frac{V_3}{4} = 1.8875 \, m/s.$

$$8 = 0.05V_2^2 + 0.4V_2^2 + \cancel{0.24}\,5.44\,V_2^2 + 0.1019V_2^2 + 0.458V_2^2$$

$$\cancel{6}.4499\,V_2^2 = 8$$

$$V_2 = \sqrt{\frac{8}{\cancel{6}.4499}} = 1.113\ m/s.$$

$$\boxed{V_2 = 1.113\ m/s}$$

Rate of Flow $= A_2V_2$

$$= \left(\frac{\pi}{4}\times 0.3^2\right)\times 1.113 = 0.0786\ m^3/sec$$

$$\boxed{Q = 0.0786\ m^3/sec.}$$

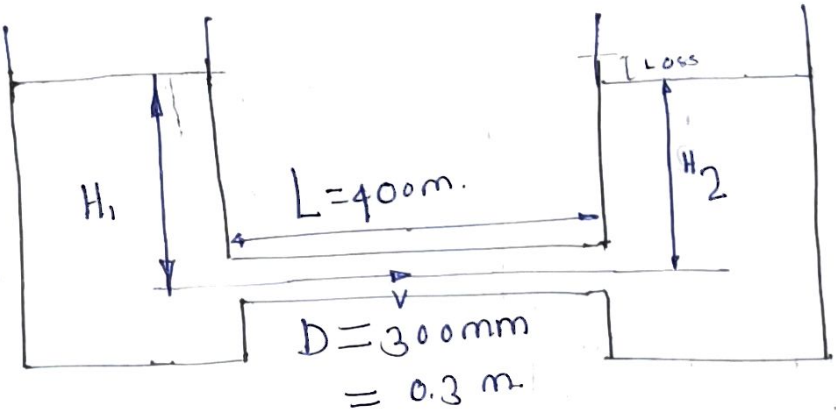
Determine the difference in elevations b/w the water surfaces in the two tanks which are connected by a horizontal pipe of diameter 300mm and length 400m. The rate of flow of water through the pipe is 300 Ltr/sec. Consider all the losses and take the value of $f = 0.008$.

$f = 0.008$

$Q = 300\ Ltr/sec$

$= 0.3\ m^3/sec.$

$A = \frac{\pi}{4}\times 0.3^2 = 0.0706\ m^2$



$H_1 = H_2 + \text{Losses}$

$$V = \frac{Q}{A} = \frac{0.3}{0.0706} = 4.25\ m/s.$$

$$\boxed{V = 4.25\ m/s}$$

Losses $= h_i + h_o + h_f$

$$h_i = \frac{0.5V^2}{2g} = \frac{0.5\times 4.25^2}{2\times 9.81} = 0.460\ m$$

$$h_o = \frac{V^2}{2g} = \frac{4.25^2}{2\times 9.81} = 0.920\ m$$

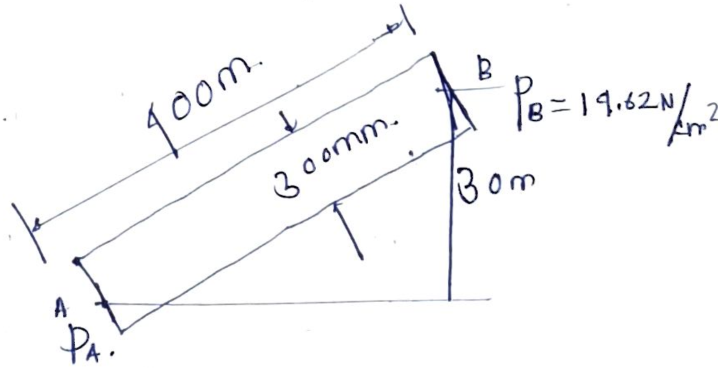
$$h_f = \frac{4fLV^2}{2gD} = \frac{4\times 0.008\times 400\times 4.25^2}{2\times 9.81\times 0.3} = 39.279\ m.$$

$h_L = 0.460 + 0.960 + 39.279$

$= 40.699m.$

$\boxed{h_L = 40.699m.}$

A pipeline AB of diameter 300mm and length 400m carries water at a rate of 50 Litres/sec. The flow takes place from A to B where point B is 30 meters above A. Find the pressure at A if the pressure at B is 19.62 N/cm^2. Take $f = 0.008$.



$D = 300mm = 0.3 m$

$f = 0.008$

$L = 400m$

$Q = 50 Ltrs/sec = 0.005 m^3/sec$

$P_B = 19.62 N/cm^2 = 19.62 \times 10^4 N/m^2$

$V = \frac{Q}{A} = \frac{0.05}{\frac{\pi}{4} \times 0.3^2}$

$= 0.707 m/s.$

$\boxed{V = 0.707 m/s.}$

$h_L = \frac{4fLV^2}{2gD} = \frac{4 \times 0.008 \times 400 \times 0.707^2}{2 \times 9.81 \times 0.3}$

$= 1.086m.$

$\boxed{h_L = 1.086m.}$

Apply Bernoulli's Equation at points A and B:

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B + h_L$$

$V_A = V_B = 0.707 m/s.$

$Z_B = 30m$

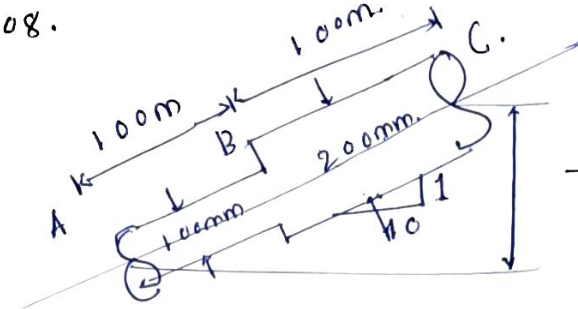
$h_L = 1.086m.$

$$\frac{P_A}{\rho g} + \frac{0.707^2}{2 \times 9.81} + 0 = \frac{19.62 \times 10^4}{1000 \times 9.81} + \frac{0.707^2}{2 \times 9.81} + 30 + 1.086$$

$$P_A = \left(\frac{19.62 \times 10^4}{1000 \times 9.81} + 30\right) \times (1000 \times 9.81) = 49.05 \times 10^4 N/m^2$$

$\boxed{P_A = 49.05 N/cm^2}$

The rate of flow of water pumped into a pipe ABC, which is 200m long, is 20 Litres/second. The pipe is laid on an upward slope of 1 in 40. The length of the portion AB is 100m and its diameter is 100mm, while the length of portion BC is also 100m but its diameter is 200mm. The change of diameter at B is sudden. The flow is taking place from A to C, where the pressure at A is $19.62 N/cm^2$ and end C is connected to a tank. Find the pressure at C and draw hydraulic gradient line and total energy line. $f = 0.008$.



$Q = 20 \text{ Litre/sec.}$

$= 0.02 m^3/sec$

$\frac{1}{40} \times 200 = 5 m.$

$D_1 = 100mm = 0.1 m.$

$A_1 = \frac{\pi}{4} \times D_1^2 = \frac{\pi}{4} \times 0.1^2 = 0.0078 m^2$

$P_1 = 19.62 N/cm^2$

$= 19.62 \times 10^4 N/m^2.$

$D_2 = 200mm = 0.2m$

$A_2 = \frac{\pi}{4} \times D_2^2 = \frac{\pi}{4} \times 0.2^2 = 0.0314 m^2.$

$V_1 = \frac{0.02}{0.0078} = 2.56 m/s.$

$V_2 = \frac{0.02}{0.0314} = 0.63 m/s.$

Applying Bernoulli's equation b/w point A and C:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

Head loss due Friction in pipe AB = $\frac{4 f L_{AB} V_1^2}{2 g D_1} = \frac{4 \times 0.008 \times 100 \times 2.56^2}{2 \times 9.81 \times 0.1}$

$= 10.688 m$

Friction in pipe BC = $\frac{4 f L_{BC} V_2^2}{2 g D_2} = \frac{4 \times 0.008 \times 100 \times 0.63^2}{2 \times 9.81 \times 0.2}$

$= 0.323 m.$

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

$$= \frac{(4.56 - 0.63)^2}{2 \times 9.81} = \cancel{0.787m} \quad 0.189m.$$

Total head loss:

$$h_L = 10.688 + 0.323 + \cancel{0.787}\ 0.189 = \cancel{11.798m}\ 11.20m.$$

$$\boxed{h_L = 11.20m}$$

$$\frac{19.62 \times 10^4}{1000 \times 9.81} + \frac{2.56^2}{2 \times 9.81} + 0 = \frac{P_2}{\rho g} + \frac{0.63^2}{2 \times 9.81} + 5 + 11.20.$$

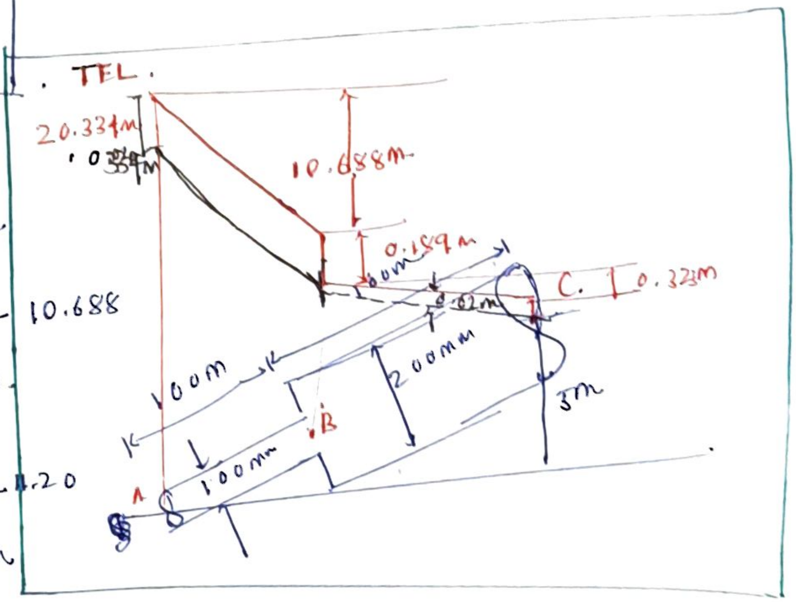
$$P_2 = \left(\frac{19.62 \times 10^4}{1000 \times 9.81} + \frac{2.56^2}{2 \times 9.81} - \frac{0.63^2}{2 \times 9.81} - 5 - 11.20 \right) \times 1000 \times 9.81$$

$$= 40.356 \times 10^3 N/m^2$$

$$= 4.035 \times 10^{4} N/m^2 = 4.035\ N/cm^2$$

$$\boxed{P_2 = 4.035\ N/cm^2}$$

HGL & TEL :



Total Energy at A = 20.334m.

Total Energy at B = 20.334 − 10.688

= 9.646m.

Total Energy at C = 20.334 − 11.20

= 9.134 m

$$\frac{V_1^2}{2g} = \frac{2.56^2}{2 \times 9.81} = 0.334m$$

Velocity Head: $\frac{V_2^2}{2g} = \frac{0.63^2}{2 \times 9.81}$

$= 0.02m.$

Pressure at summit:

Apply Bernoulli's Theorem b/w points A and C.

$$\frac{P_A}{\rho g}+\frac{V_A^2}{2g}+Z_A=\frac{P_C}{\rho g}+\frac{V_C^2}{2g}+Z_C+h_L$$ (Loss of head due to friction b/w A and C).

$$0+0+\overset{0}{Z_A}=\frac{P_C}{\rho g}+\frac{V^2}{2g}+3+h_L$$

$$h_L=\frac{4fLV^2}{2gD}$$

$$=\frac{4\times0.005\times100\times2.801^2}{2\times9.81\times0.2}$$

$$\frac{P_C}{\rho g}+\frac{V^2}{2g}+3+h_L=0$$

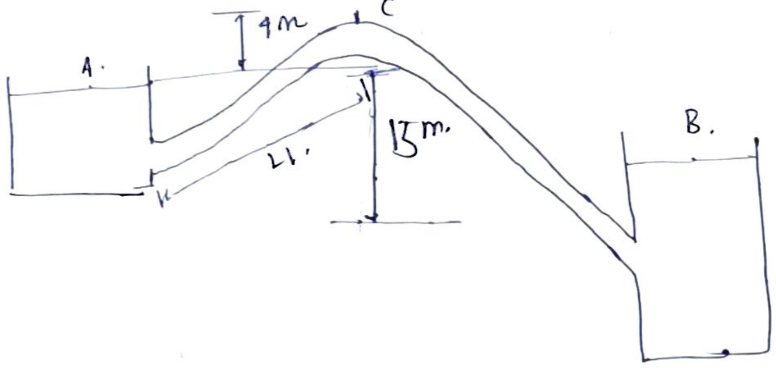
$$\frac{P_C}{\rho g}=-\frac{V^2}{2g}-3-h_L$$

$$P_C=-\left(\frac{2.801^2}{2\times9.81}+3+3.998\right)\times1000\times9.81 \quad =3.998m$$

$$=-7.257\times10^4\ N/m^2$$

$$\boxed{P_C=-7.257N/cm^2}$$

A syphon of diameter 200mm connects two reservoirs having a difference in elevation of 15m. Total length of the syphon is 600m and the summit is 4m above the water level in the upper reservoir. If the separation takes place at 2.8m of water absolute. Find the maximum length of the syphon from upper reservoir to the summit. Take $f=0.004$ and atmospheric pressure $=10.3m$ of water.



$D=200mm=0.2m.$

$H=15m.$

$L=600m.$

$f=0.004$

$P_{atm}=10.3m$ of water.

A syphon of diameter 200mm connects two reservoirs having a difference in elevation of 20m. The length of the syphon is 500m and the summit is 3.0m above the water level in the upper reservoir. The length of the pipe from upper reservoir to summit is 100m. Determine the discharge through the syphon and also pressure at the summit. Neglect minor losses. The coefficient of friction $f = 0.005$.

$D = 200\,mm = 0.2\,m$

$f = 0.005$

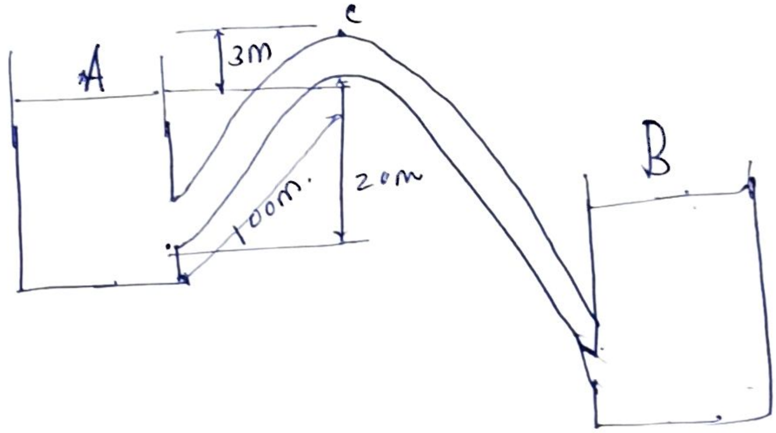
$L = 500\,m.$

$H = 20\,m.$

$h = 3\,m$

$L_1 = 100\,m.$ $f = 0.005$

(Length of the syphon up to summit).



If minor losses are neglected then the loss of head takes place only due to friction.

Apply Bernoulli's eq^n to points A & B.

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B + h_L$$ (Loss of head due to friction from A to B).

$P_A = P_B =$ Atmospheric pressure

$V_A = V_B = 0.$

$$0 + 0 + Z_A = 0 + 0 + Z_B + h_L.$$

$$Z_A - Z_B = h_L.$$

$$20 = \frac{4fLV^2}{2gD}.$$

$$V = \sqrt{\frac{20 \times 2 \times 9.81 \times 0.2}{4 \times 0.005 \times 500}} = 2.801\,m/s.$$

$$\boxed{V = 2.801\,m/s}$$

$$Q = AV = 2.801 \times \left(\frac{\pi}{4} \times 0.2^2\right) = 0.0879\,m^3/sec \quad \boxed{Q = 0.0879\,m^3/sec}$$

At sec. 1

$$TEL = \frac{P_1}{\rho g} + \frac{v_1^2}{2g} + Z_1 = \frac{54.105\times10^4}{950\times9.81} + \frac{0.744^2}{2\times9.81} + 0 = 58.083\,m$$

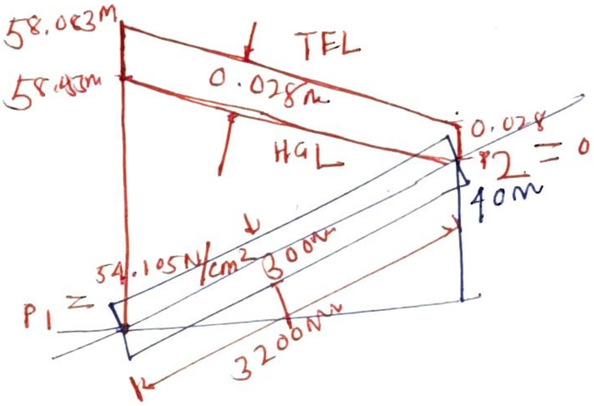
$$HGL = 58.083 - \frac{v^2}{2g}$$

$$= 58.083 - \frac{0.744^2}{2\times9.81} = 58.055\,m.$$

At sec. 2

$$TEL = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 = 0 + \frac{0.744^2}{2\times9.81} + 40 = 40.028\,m$$

HGL = 40 m



A pipeline 300mm in diameter and 3200m long is used to pump of 50 kg/sec of an oil whose density is 950 kg/m^3 and whose kinematic viscosity is 2.1 Stokes. The center of the pipeline at upper end is 40m above than that at the lower end. The discharge at upper end is atmospheric. Find the pressure at the lower end and draw the HGL and TEL.

$D = 300\,mm = 0.3\,m.$

$L = 3200\,m$

$\rho = 950\,kg/m^3$

$\nu = 2.1\,stokes = 2.1\times10^{-4}\ m^2/sec$

Mass flow rate $= 50$ kg/sec

$= \rho \cdot AV.$

$= \rho Q.$

$Q = \frac{50}{950} = 0.0526\,m^3/sec.$

$V = \frac{0.0526}{\frac{\pi}{4}\times0.3^2} = 0.744\,m/sec.$

$Re = \frac{VD}{\nu}$

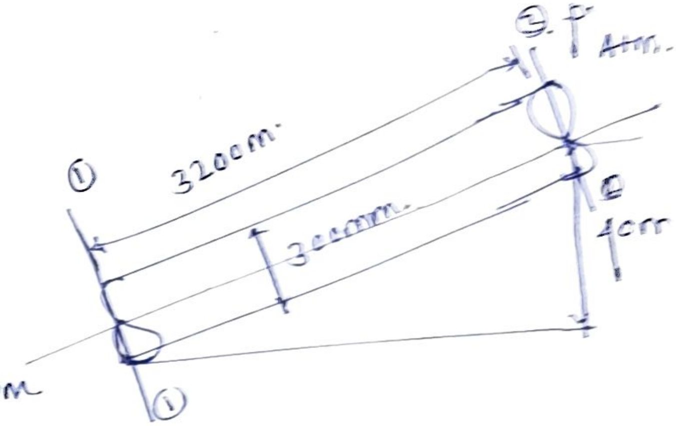
$= \frac{0.744\times0.3}{2.1\times10^{-4}} = 1062.85 < 2000$

$\boxed{Re = 1062.85}$ The flow is laminar.

$f = \frac{16}{Re} = \frac{16}{1062.85}$

$= 0.015$

$\boxed{f = 0.015}$



Applying Bernoulli's Theorem at sec. 1 & 2

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L \qquad h_L = \frac{4fLv^2}{2gD} = \frac{4\times0.015\times3200\times0.744^2}{2\times9.81\times0.3}$$

$= 18.056\,m$

$$\frac{P_1}{950\times9.81} + 0 = 0 + 40 + 18.056$$

$v_1 = v_2 \quad P_2 = 0$

$z_1 = 0$

$z_2 = 40\,m$

$P_1 = 58.056\times950\times9.81$

$= 54.105\times10^4\ N/m^2 = 54.105\ N/cm^2$

Applying Bernoulli's equation to points A & C.

$$\frac{P_A}{\rho g}+\frac{V_A^2}{2g}+Z_A=\frac{P_C}{\rho g}+\frac{V_C^2}{2g}+Z_C+h_L$$

$$10.3+0+0=2.8+\frac{V_C^2}{2g}+4+h_L.$$

$$10.3=2.8+4+\frac{V_C^2}{2g}+h_L$$

$$h_L=3.5\frac{V_C^2}{2g}$$

$$\boxed{h_L=3.5-\frac{V_C^2}{2g}.}$$

Applying Bernoulli's equation to points A and B.

$$\frac{P_A}{\rho g}+\frac{V_A^2}{2g}+Z_A=\frac{P_B}{\rho g}+\frac{V_B^2}{2g}+Z_B.$$

$P_A=P_B=$ Atmospheric pressure.

$Z_A=15m.$

$Z_B=0.$

$V_A=0$ $V_B=0$

$$0+0+15=0+0+0+h_L$$

$$h_L=15$$

$$\frac{4fLv^2}{2gD}=15.$$

$$V=\sqrt{\frac{2\times9.81\times0.2\times15}{4\times0.004\times600}}$$

$$=2.476\ m/s.$$

$$\boxed{V=2.476\ m/s.}$$

$$h_L = 3.5 - \frac{V_c^2}{2g}$$

$$= 3.5 - \frac{2.476^2}{2 \times 9.81} = 3.1875 m.$$

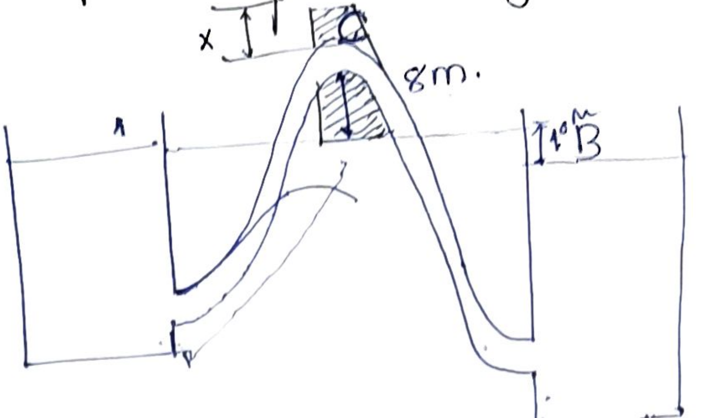
$$\frac{4 f L_1 V_1^2}{2gD} = 3.1875$$

$$L_1 = \frac{3.1875 \times 2 \times 9.81 \times 0.2}{4 \times 0.004 \times 2.476^2}.$$

$$= 127.514 m$$

$$\boxed{L_1 = 127.514 m}$$

A Syphon of diameter 200mm connects two reservoirs whose water surface level different by 40m. The total length of the pipe is 8000m. The pipe crosses a ridge. The summit of ridge is 8m above the level of water in the upper reservoir. Determine the minimum depth of the pipe below the summit of ridge, if the absolute pressure head at the summit of syphon is not to fall below 3.0m of water. Take f = 0.006 and atmospheric pressure head = 10.3 of water. The length of syphon from the upper reservoir to the summit is 500m. Find the discharge.



$D = 200mm = 0.2m.$

$H = 40m.$

$L = 8000m.$

$f = 0.006$

$L_1 = 500m.$

$\frac{P_A}{\rho g} = 10.3\ m$ (atmospheric pressure head)

$\frac{P_C}{\rho g} = 3\ m$

Apply Bernoulli's Equation b/w A and B. Taking datum line passing through point B.

$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B + h_L$ (head loss due to friction b/w A and B.)

$0 + 0 + 40 = 0 + 0 + 0 + \frac{4fLV^2}{2gD}.$

$\frac{4 \times 0.006 \times 8000 \times V^2}{2 \times 9.81 \times 0.2} = 40$

$V = \sqrt{\frac{40 \times 2 \times 9.81 \times 0.2}{4 \times 0.006 \times 8000}} = 0.904\ m/s$

$\boxed{V = 0.904\ m/s.}$

Applying Bernoulli's equation to points b/w A and C.

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B + h_L \quad \text{(headloss due to friction b/w A \& C)}$$

$$10.3 + 0 + 0 = 3 + \frac{v^2}{2g} + (8-x) + \frac{4fLv^2}{2gD}$$

$$10.3 = 3 + \frac{0.904^2}{2\times9.81} + (8-x) + \frac{4\times0.006\times500\times0.904^2}{2\times9.81\times0.2}$$

$$8 - x = 4.75$$

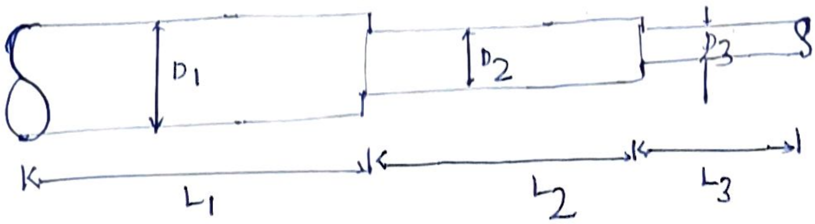
$$x = 8 - 4.75$$

$$= 3.25m$$

$$\boxed{x = 3.25m.}$$

$$Q = \left(\frac{\pi}{4}\times D^2\right)\times v = \left(\frac{\pi}{4}\times 0.2^2\right)\times 0.904 = 0.0283 \ m^3/sec$$

$$\boxed{Q = 0.0283 \ m^3/sec.}$$



$L_1 = 800m$ $\quad\quad$ $D_1 = 500mm$

$L_2 = 500m$ $\quad\quad$ $D_2 = 400mm$

$L_3 = 400m$ $\quad\quad$ $D_3 = 300mm.$

$$L = L_1 + L_2 + L_3$$
$$= 800 + 500 + 400$$
$$= 1700m.$$

$$\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}$$

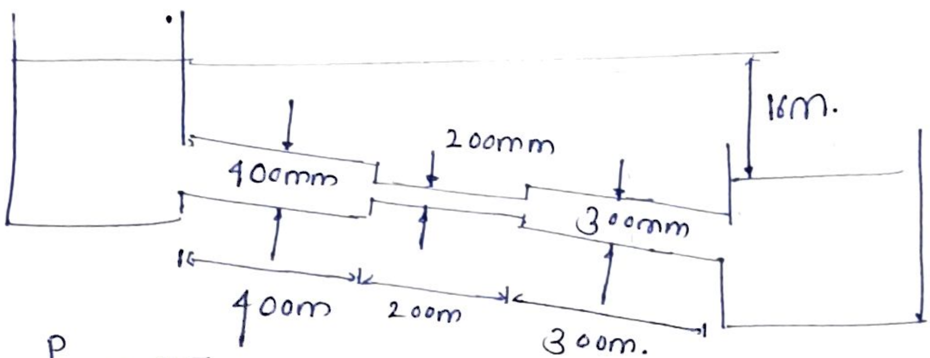
$$D^5 = \frac{L}{\left(\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}\right)}$$

$$D = \left[\frac{800+500+400}{\left(\frac{800}{0.5^5} + \frac{500}{0.4^5} + \frac{300}{0.3^5}\right)}\right]^{1/5}$$

$$= 0.372m.$$

$$\boxed{D = 372mm.}$$

Three pipes of 400mm, 200mm and 300mm diameters have lengths of 400m, 200m and 300m respectively. They are connected in series to make a compound pipe. The ends of ~~two~~ compound pipes are connected with two tanks whose difference of water levels is 16m. If coefficient of friction of these pipes is same and equal to 0.005. Determine the discharge through the compound pipe neglecting first the minor losses and then including them.



$D_1 = 400mm$
$= 0.4m.$

$D_2 = 200mm$
$= 0.2m.$

$D_3 = 300mm$
$= 0.3m.$

$f = 0.005$

(i) Neglecting minor losses.

$$H = \frac{4fL_1V_1^2}{2gD_1} + \frac{4fL_2V_2^2}{2gD_2} + \frac{4fL_3V_3^2}{2gD_3}$$

From continuity Eqn:

$A_1V_1 = A_2V_2 = A_3V_3.$

$\left(\frac{\pi}{4}\times 0.4^2\right)V_1 = \left(\frac{\pi}{4}\times 0.2^2\right)V_2 = \left(\frac{\pi}{4}\times 0.3^2\right)V_3$

$$\boxed{\begin{aligned} V_2 &= 4V_1 \\ V_3 &= 1.78V_1 \end{aligned}}$$

$$16 = \frac{4\times 0.005\times 400\times V_1^2}{2\times 9.81\times 0.4} + \frac{4\times 0.005\times 200\times (4V_1)^2}{2\times 9.81\times 0.2} + \frac{4\times 0.005\times 300\times (1.78V_1)^2}{2\times 9.81\times 0.3}$$

$16 = 1.019V_1^2 + 16.309V_1^2 + 3.229V_1^2$

$$V_1 = \sqrt{\frac{16}{20.557}}$$

$= 0.882$ m/s.

$\boxed{V_1 = 0.882 \text{ m/s.}}$

$V_2 = 4\times 0.882 = 3.528$ m/s

$V_3 = 1.78\times 0.882 = 1.569$ m/s.

$Q = A_1V_1$

$= \left(\frac{\pi}{4}\times 0.4^2\right)\times 0.882$

$= 0.1108\ m^3/sec$

$\boxed{Q = 0.1108\ m^3/sec.}$

$$\frac{\left(2.25v_1^2 - 0.5625v_1^2\right)}{2g} + \frac{4\times 0.0048\times 210\times\left(0.5625v_1\right)^2}{2g\times 0.4} + \frac{\left(0.5625v_1\right)^2}{2g.}$$

$$12 = \frac{v_1^2}{2g.}\left[0.5 + 20 + 2.531 + 89.505 + (1.6875)^2 + 3.189 + 0.316\right]$$

$$v_1 = \sqrt{\frac{2\times 9.81\times 12}{118.888}}$$

$$= 1.407 \text{ m/s.}$$

$$\boxed{v_1 = 1.407 \text{ m/s.}}$$

Rate of flow: $Q = A_1 v_1$

$$= \left(\frac{\pi}{4}\times 0.3^2\right)\times 1.407$$

$$= 0.09945 \text{ m}^3/\text{sec}$$

$$\boxed{Q = 0.09945 \text{ m}^3/\text{sec}}$$

(II) Neglecting minor losses:

$$H = \frac{4f_1 L_1 v_1^2}{2gD_1} + \frac{4f_2 L_2 v_2^2}{2gD_2} + \frac{4f_3 L_3 v_3^2}{2gD_3}$$

$$12 = \frac{4\times 0.005\times v_1^2\times 300}{2\times 9.81\times 0.3} + \frac{4\times 0.0052\times 170\times(2.25v_1)^2}{2\times 9.81\times 0.2} + \frac{4\times 0.0048\times 210\times(0.5625v_1)^2}{2\times 9.81\times 0.4}$$

$$12 = 1.019v_1^2 + 4.561v_1^2 + 0.162v_1^2$$

$$v_1 = \sqrt{\frac{12}{5.742}} = 1.445$$

$$\boxed{v_1 = 1.445 \text{ m/s.}}$$

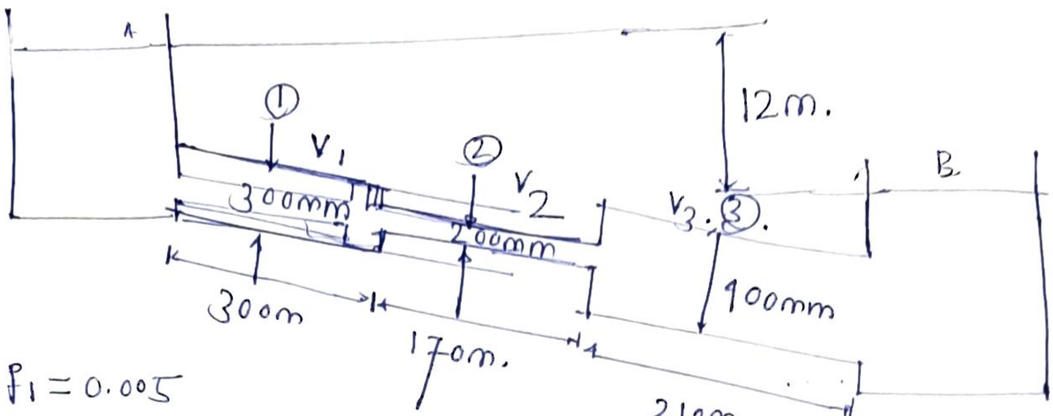
$$Q = \left(\frac{\pi}{4}\times 0.3^2\right)\times 1.445$$

$$= 0.1021 \text{ m}^3/\text{sec}$$

The difference in water surface levels in two tanks, which are connected by three pipes in series of lengths 300m, 170m, 210m and diameter 300mm, 200mm and 400mm respectively is 12m. Determine the rate of flow of water if coefficient of friction are 0.005, 0.0052 and 0.0048 respectively

Considering minor losses also

(ii) Neglecting minor losses also.



$f_1 = 0.005$

$f_2 = 0.0052$

$f_3 = 0.0048.$

$D_1 = 300mm = 0.3m.$

$D_2 = 200mm = 0.2m$

$D_3 = 400mm = 0.4m$

$L_1 = 300m$

$L_2 = 170m$

$L_3 = 210m$

From continuity Eqn: $A_1V_1 = A_2V_2 = A_3V_3.$

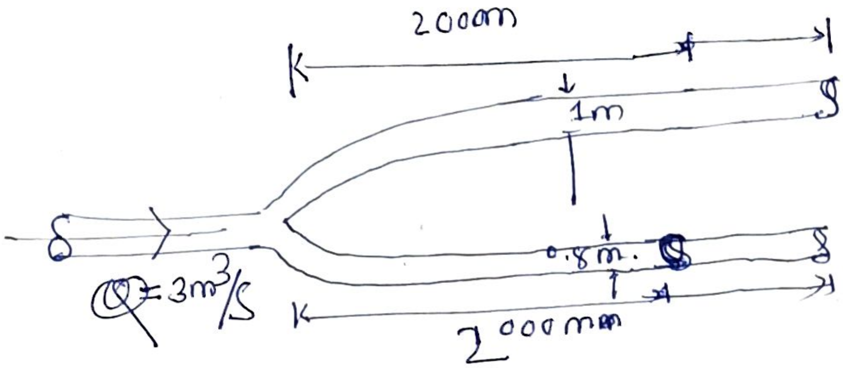
$$\left(\frac{\pi}{4}\times 0.3^2\right)V_1 = \left(\frac{\pi}{4}\times 0.2^2\right)V_2 = \left(\frac{\pi}{4}\times 0.4^2\right)V_3$$

$$\boxed{\begin{aligned} V_2 &= 2.25V_1 \\ V_3 &= 0.5625V_1. \end{aligned}}$$

(i). The difference in liquid surface levels is equal to sum of the total head loss in the pipes:

$$H = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{2gD_1} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{2gD_2} + \frac{(V_2-V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{2gD_3} + \frac{V_3^2}{2g}$$

$$= \frac{0.5V_1^2}{2g} + \frac{4\times0.005\times300V_1^2}{2g\times0.3} + \frac{0.5\times(2.25V_1)^2}{2g} + \frac{4\times0.0052\times170\times(2.25V_1)^2}{2g\times0.2}$$



$Q = 3\,m^3/sec.$

$L_1 = 2000m \qquad D_1 = 1000mm = 1m$

$L_2 = 2000m \qquad D_2 = 800mm = 0.8m.$

$f = 0.005$

Loss of head for each branchrire is same.

$h_{f_1} = h_{f_2}.$

$$\frac{4 f_1 L_1 v_1^2}{2 g D_1} = \frac{4 f_2 L_2 v_2^2}{2 g D_2}. \qquad f_1 = f_2 = 0.005, \quad L_1 = L_2 = 2000m.$$

$$\frac{v_1^2}{2 \times 9.81 \times 1} = \frac{v_2^2}{2 \times 9.81 \times 0.8}$$

$$v_1 = \sqrt{\frac{1}{0.8}} \times v_2$$

$$\boxed{v_1 = 1.118 v_2.}$$

$$Q_1 = A_1 v_1 = \left(\frac{\pi}{4} \times 1^2\right) \times 1.118 v_2 = 0.878 v_2.$$

$$Q_2 = A_2 v_2 = \left(\frac{\pi}{4} \times 0.8^2\right) \times v_2 = 0.502 v_2.$$

$Q = Q_1 + Q_2$

$3 = 0.878 v_2 + 0.502 v_2 = 1.38 v_2$

$$v_2 = \frac{3}{1.38} = 2.173 \qquad \boxed{v_2 = 2.173\,m/s.}$$

$v_1 = 1.118 \times 2.173$

$= 2.429\,m/s.$

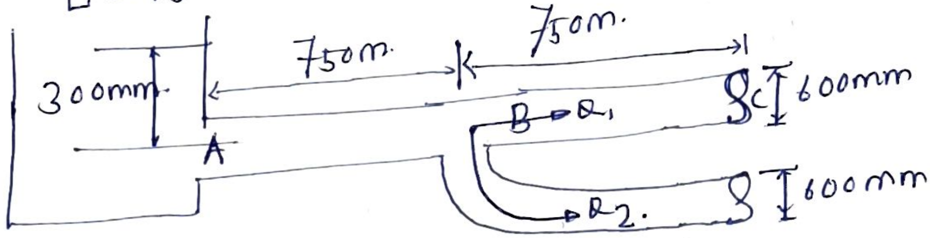
$$Q_1 = \left[\frac{\pi}{4} \times (1)^2\right] \times 2.429 = 1.907 \ m^3/sec.$$

$$Q_2 = \left[\frac{\pi}{4} \times 0.8^2\right] \times 2.173 = 1.092 \ m^3/sec$$

A pipe line of 0.6 m diameter is 1.5 km long. To increase the discharge another line of same diameter is introduced parallel to the first in the second half of length. Neglecting the minor losses, find the increase in discharge if 4f = 0.004. The head at inlet is 300mm.

$D = 600mm$ $\qquad$ $4f = 0.004$

$L = 1500m$ $\qquad$ $f = \frac{0.004}{4} = 0.01.$



(1) Discharge in 1500m pipe:

$$h_f = \frac{4fLv^2}{2gd}.$$

$$0.3 = \frac{0.004 \times 1500 \times v^2}{2 \times 9.81 \times 0.6}$$

$$v = \cancel{0.767} \ m/s. \quad \cancel{6.485} \quad 0.242.$$

$$\boxed{V = 0.242 \ m/s.}$$

$$Q = AV = \left(\frac{\pi}{4} \times 0.6^2\right) \times \cancel{0.767} \ 0.242$$

$$= 0.\cancel{2168}\,0684 \ m^3/sec.$$

$$\boxed{Q = 0.0684 \ m^3/sec.}$$

$Q = Q_1 + Q_2$

$\Rightarrow Q_1 = Q_2 = \frac{Q}{2}$.

Head Loss through ABC = Head Loss through AB + head Loss through BC.

$(h_L)_{AB} = \frac{4fL_1 V_{AB}^2}{2gD} = \frac{4 \times 0.01 \times 750 \times V_{AB}^2}{2 \times 9.81 \times 0.6}$

$Q_{AB} = A \times V_{AB}$.

$V_{AB} = \frac{4Q}{\pi D^2}$.

$\boxed{V_{AB} = \frac{4Q}{\pi \times 0.6^2}}$.

$= \frac{4 \times 0.01 \times 750 \times \left(\frac{4Q}{\pi \times 0.6^2}\right)^2}{2 \times 9.81 \times 0.6}$

$= 31.877 Q^2$.

$(h_L)_{BC} = \frac{4fL_2 V_{BC}^2}{2gD} = \frac{4 \times 0.01 \times 750 \times \left(\frac{4Q}{2\pi \times 0.6^2}\right)^2}{2 \times 9.81 \times 0.6}$

$Q_{BC} = \frac{Q_{AB}}{2}$

$= \frac{4Q}{2\pi \times 0.6^2}$

$= 7.969 Q^2$

$0.3 = 31.877 Q^2 + 7.969 Q^2$

$Q = \sqrt{\frac{0.3}{31.877 + 7.969}} = 0.0867\ m^3/sec.$

$\boxed{Q = 0.0867\ m^3/sec.}$

Increase in discharge $= Q - Q^*$

$= 0.0867 - 0.0685$

$= 0.0182\ m^3/sec$

ORIFICES AND MOUTHPIECES

(I) Orifice is a small opening of any cross-section on the side or at the bottom of the tank, through which a fluid is flowing.

(II) A mouthpiece is a small length of a pipe which is two or three times its diameter in length, fitted in a tank or vessel containing the fluid.

(III) Orifices as well as mouthpieces are used for measuring the rate of flow of fluid.

CLASSIFICATION OF ORIFICES

The orifices are classified as on the basis of their size, shape, nature of discharge ~~and~~ shape of the upstream edge.

1. The orifices are classified as small or large orifices depending upon the size of the orifice and head of the liquid from the center of orifice.

If the head of the liquid from the center of the orifice is more than five times the depth of orifice the orifice is called small orifice.

If the head of the liquid is ~~more~~ less than five times the depth of orifice is known as large orifice.

2. The orifices are classified as depending upon their cross sectional areas.
   - (I) circular orifice.
   - (II) Triangular orifice.
   - (III) Rectangular orifice.
   - (IV) Square orifice.

3. The orifices are classified as depending upon their nature of discharge
   - (I) Free discharging orifice
   - (II) Drowned or submerged orifices.

Submerged orifice — fully submerged orifice — partially submerged orifice.

$C_v = \frac{V_{Act.}}{V_{Th.}}$

$V_{Act} = 0.98 \times 14 = 13.72\ m/s.$

$C_d = 0.6$

$= \frac{Q_{Act.}}{Q_{Th.}}$

$Q_{Act} = C_d \times Q_{Th.}$

$= 0.6 \times (V_{Th} \times A_0).$

$= 0.6 \times (14 \times 1.25 \times 10^{-3})$

$= 10.5 \times 10^{-3}\ m^3/sec.$

$\boxed{Q_{Act} = 10.5 \times 10^{-3}\ m^3/sec.}$

The head of water over the center of an orifice of diameter 20mm is 1m. The actual discharge through the orifice is 0.85 Litres/sec. Find the coefficient of discharge.

$D = 20mm = 0.02m$ $A = \frac{\pi}{4} \times 0.02^2$

$H = 1m.$ $= 3.1415 \times 10^{-4}\ m^2$

$Q_{Act.} = 0.85 \times 10^{-3}\ m^3/sec.$

$V_{Th.} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 1}$

$= 4.429\ m/s.$

$\boxed{V_{Th.} = 4.429\ m/sec.}$

$Q_{Th.} = (3.1415 \times 10^{-4}) \times 4.429.$

$= 1.391 \times 10^{-3}\ m^3/sec.$

$\boxed{Q_{Th.} = 1.391 \times 10^{-3}\ m^3/sec}$

$C_d = \frac{Q_{Act.}}{Q_{Th.}}$

$= \frac{0.85 \times 10^{-3}}{1.391 \times 10^{-3}}$

$= 0.611$

$\boxed{C_d = 0.611}$

3. Co-efficient of discharge: (C_d)

It is the ratio of the actual discharge from an orifice to the theoritical discharge from the orifice.

$$\boxed{C_d = \frac{Q_{act.}}{Q_{Th.}} = \frac{\text{Actual discharge}}{\text{Theoritical discharge}}}$$

$$C_d = \frac{V \cdot A_C}{V_{Th.} \times A_O} = \frac{\text{Actual velocity} \times \text{Actual Area}}{\text{Theoritical velocity} \times \text{Theoritical Area}}$$

$$= C_v \times C_c$$

$$\boxed{C_d = C_v \times C_c}$$

The value of Cd varies from 0.61 to 0.65.

$$\boxed{C_d = 0.62}$$

1. The head of water over an orifice of diameter 40mm is 10m. Find the actual discharge and actual velocity of the jet at vena-contracta.

$C_d = 0.6$

$C_v = 0.98$

$D = 40mm = 0.004m.$

$H = 10m.$

Theoritical velocity: $V_{Th.} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 10}$

$= 14 \ m/sec.$

$A_o = \frac{\pi}{4} \times 0.04^2 = 1.25 \times 10^{-3} \ m^2.$

HYDRAULLIC COEFFICIENTS :

The hydraulic coefficients are
1. co-efficient of velocity : C_v
2. co-efficient of contraction : C_c
3. co-efficient of discharge : C_d

1. co-efficient of velocity : (C_v).

It is defined as the ratio b/w the actual velocity of a jet of liquid at vena-contracta and the theoritical velocity of jet.

$$C_v = \frac{\text{Actual velocity of jet at vena-contracta}}{\text{Theoritical velocity}}$$

$$= \frac{V}{\sqrt{2gH}}$$

$$\boxed{C_v = \frac{V}{\sqrt{2gh}}}$$

$V_{Th.} = \sqrt{2gH}$ = Theoritical velocity

$V_{Act.}$ = Actual velocity

The value of C_v varies from $0.95 - 0.99$.

$C_v = 0.98$ taken for sharp edged orifices.

2. co-efficient of contraction : (C_c)

coefficient of contraction is defined as the ratio of the area of the jet at vena-contracta to area of the orifice.

$$C_c = \frac{\text{Area of jet at vena-contracta}}{\text{Area of jet at orifice}}$$

$$= \frac{A_c}{A_o}$$

$$\boxed{C_c = \frac{A_c}{A_o}}$$

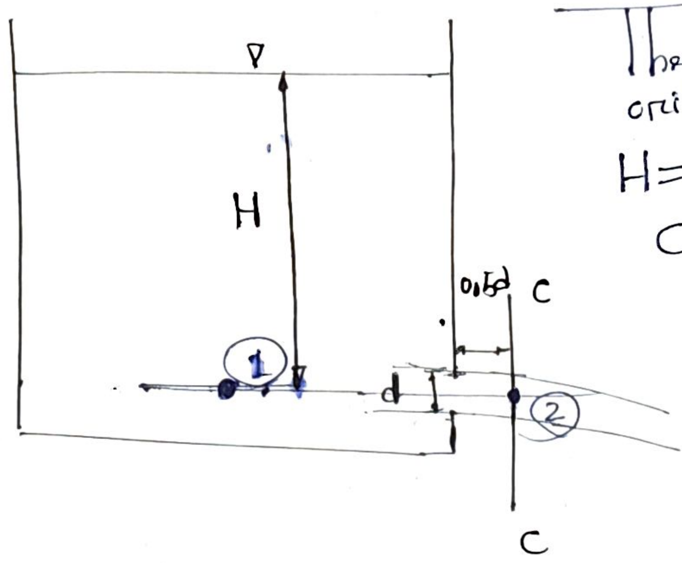
The value of C_c varies from $0.61 - 0.69$.

generally taken $\boxed{C_c = 0.64}$

4. Depending upon shape of upstream edge: (I) Sharp-edged orifice

(II) Bell-mouthed orifice.

FLOW THROUGH AN ORIFICE:



The tank is fitted with a circular orifice.

H = Head of the liquid above the center of orifice.

(I) The section is approximately at a distance of half of diameter of the orifice.

(II) At this section, the streamlines are straight and parallel to each other and perpendicular to the plane of the orifice.

(III). The section is called vena-contracta.

Apply Bernoulli's Theorem at sec. 1 & 2.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$Z_1 = Z_2$

$V_1 << V_2$. so neglected

$$H + 0 = 0 + \frac{V_2^2}{2g}$$

$H = \frac{P_1}{\rho g}$

P_2 - Atmospheric pressure

$$\frac{V_2^2}{2g} = H \qquad V_2 = \sqrt{2gH}$$

$$\boxed{V_2 = \sqrt{2gH}}$$

- This is theoretical velocity.
- Actual velocity will be less than this value.

The water is allowed to flow through an orifice fitted to a tank under a constant head H.

The water is collected in a measuring tank for a known time 't'.

The height of water in the measuring tank is noted.

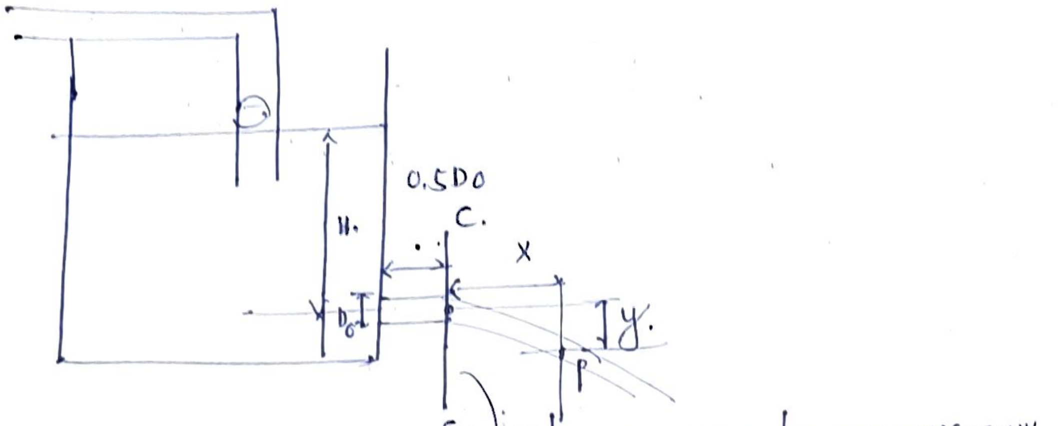
Actual discharge through orifice:

$$\boxed{Q_{act} = \frac{\text{Area of measuring tank} \times \text{Height of water in measuring tank}}{\text{Time}(t)}}$$

Theoritical discharge: $Q_{Th.} = \text{Area of orifice} \times \sqrt{2gH}$

$= A_0 \times \sqrt{2gH}$

$$\boxed{Q_{Th.} = A_0 \times \sqrt{2gH}}$$



$$\boxed{C_d = \frac{Q}{A_0 \times \sqrt{2gH}}}$$

x = horizontal distance travelled by the particle in time 't'.

y = vertical distance b/w P and c/c

V = actual velocity of jet at vena contracta.

Horizontal distance: $x = vt \Rightarrow \boxed{t = \frac{x}{v}}$

Vertical distance: $y = \frac{1}{2} g t^2$

$$= \frac{1}{2} g \left(\frac{x}{v}\right)^2$$

$$= \frac{g x^2}{2 v^2}$$

$$v = \sqrt{\frac{g x^2}{2y}}$$

$$\boxed{V_{Act.} = \sqrt{\frac{g x^2}{2y}}}$$

Theoritical velocity: $\boxed{V_{Th.} = \sqrt{2gH}}$

Coefficient of velocity: $C_v = \frac{V_{Act.}}{V_{Th}}$

$$= \frac{\sqrt{g x^2 / 2y}}{\sqrt{2gH}}$$

$$= \sqrt{\frac{g x^2}{4 g H y}} = \frac{x^2}{4yH}$$

$$\boxed{C_v = \sqrt{\frac{x^2}{4yH}}}$$

Coefficient of contraction: $\boxed{C_c = \frac{C_d}{C_v}}$

A jet of water issuing from a sharp edged vertical orifice under a constant head of 10 cm at a certain point, has the vertical and horizontal coordinates measured from vena contracta as 10.5 cm and 20 cm respectively. Find the value of C_v also find the value of C_c if $C_d = 0.60$.

$H = 10cm = 0.1m.$

$y = 10.5cm. = 0.105m.$

$X = 20cm = 0.2M$

$C_d = 0.60$

$$C_v = \frac{\sqrt{2gH}}{\sqrt{\frac{gx^2}{2y}}} = \frac{\sqrt{2 \times 9.81 \times 0.1}}{\sqrt{\frac{9.81 \times 0.2^2}{2 \times 0.105}}}$$

$$= 1.024$$

$$\boxed{C_v = 1.024.}$$

$C_d = C_v \times C_c.$

$$C_c = \frac{C_d}{C_v} = \frac{0.6}{1.024} = 0.585$$

$$\boxed{C_c = 0.585}$$

The head of water over an orifice of diameter 100mm is 10m. The water coming out of the orifice is collected in a circular tank of diameter 1.5m. The rise of water level in this tank is 1m in 25 seconds. Also the coordinates of a point on the jet, measured from vena contracta are 4.3m horizontal and 0.5m vertical. Find the coefficients of Cd, Cv and Cc.

$D_0 = 100mm = 0.1m$

$H = 10m.$

$D_{Tank} = 1.5m.$

$t = 25$ seconds

$H_w = 1m.$

$x = 4.3m$

$y = 0.5m.$

Actual Discharge: $Q_{Act} = \dfrac{\text{Area of tank} \times \text{Height of water}}{\text{Time}(t)}$

$= \dfrac{\left(\frac{\pi}{4} \times 1.5^2\right) \times 1}{25}$

$= 0.0706\ m^3/sec$

$$\boxed{Q_{Act.} = 0.0706\ m^3/sec}$$

Theoritical velocity: $V_{Th.} = \sqrt{2gH}$

$= \sqrt{2 \times 9.81 \times 10} = 14m$

$$\boxed{V_{Th.} = 14m.}$$

Actual velocity: $C_{Act.} = \sqrt{\dfrac{gx^2}{2y}} = \sqrt{\dfrac{9.81 \times 4.3^2}{2 \times 0.5}} = 13.46m^3/sec$

$V_2 = 11.205\ m/s.$

$$\boxed{V_2 = 11.205\ m/s.}$$

Theoritical discharge: $Q_{Th} = A_2 V_2$

$= \left(\frac{\pi}{4} \times 0.05^2\right) \times 11.205 = 0.022\ m^3/sec.$

$$\boxed{Q_{Th} = 0.022\ m^3/sec}$$

$$C_d = \frac{0.02}{0.022}$$

$= 0.909$

$$\boxed{C_d = 0.909.}$$

(II) Loss of head due to fluid resistance:

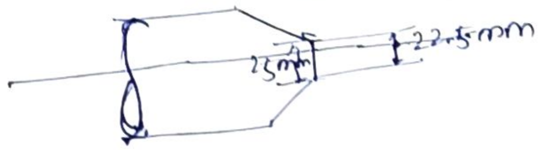
Applying Bernoulli's equation at the outlet of nozzle and to the jet of water.

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2+h_L.$$

$\frac{P_1}{\rho g}=\frac{P_2}{\rho g}$ = Atmospheric pressure head.

$Z_1=Z_2$.

$$\frac{V_1^2}{2g}=\frac{V_2^2}{2g}+h_L.$$



$V_1=V_{Th.}=\sqrt{2gH}$

$V_2=V_{Act.}=C_V\sqrt{2gH}$

$$\frac{(\sqrt{2gH})^2}{2g}=\frac{(C_V\sqrt{2gH})^2}{2g}+h_L.$$

$$h_L=\frac{(\sqrt{2gH})^2}{2g}-\frac{(C_V\sqrt{2gH})^2}{2g}$$

$$=\frac{(\sqrt{2gH})^2\left[1-(C_V)^2\right]}{2g}$$

$$=\frac{2gH(1-C_V^2)}{2g}$$

$$=H(1-C_V^2).$$

$$h_L=H(1-C_V^2)$$

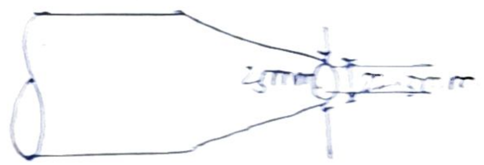
$$=60(1-0.925^2)$$

$$=8.6625\,m.$$

$$\boxed{h_L=8.6625\,m.}$$

$D_0 = 25\,mm$

$D_c = 22.5\,mm$



$Q_{act} = \frac{0.76}{60}$

$= 0.0126\,m^3/sec.$ $H = 60\,m.$

coefficient of contraction: $\frac{A_c}{A_0}$

$$C_c = \frac{\frac{\pi}{4} \times D_c^2}{\frac{\pi}{4} \times D_0^2}$$

$$= \frac{\frac{\pi}{4} \times (22.5)^2}{\frac{\pi}{4} \times (25)^2} = 0.81.$$

$\boxed{C_c = 0.81}$

$V_{Th.} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 60}$

$= 34.310\,m/s.$

$\boxed{V_{Th.} = 34.310\,m/s.}$

$Q_{Th} = A_0 . V_{Th} = \frac{\pi}{4} \left(\frac{25}{1000}\right)^2 \times 34.310 = 0.0168\,m^3/sec$

$\boxed{Q_{Th} = 0.0168\,m^3/sec}$

$C_d = \frac{Q_{Act.}}{Q_{Th}} = \frac{0.0126}{0.0168}$

$= 0.75$

$\boxed{C_d = 0.75}$

$C_d = C_v \times C_c \Rightarrow C_v = \frac{0.75}{0.81} = 0.925$ $\boxed{C_v = 0.925}$

Theoritical discharge: $Q_{Th.} = A_0 \times \sqrt{2gH}$

$= \left(\frac{\pi}{4} \times 0.1^2\right) \times 14 = 0.1099\ m^3/sec.$

$$\boxed{Q_{Th.} = 0.1099\ m^3/sec.}$$

$$C_d = \frac{Q_{ACT.}}{Q_{Th.}} = \frac{0.0706}{0.1099}$$

$= 0.642$

$$\boxed{C_d = 0.642}$$

$$C_V = \frac{V_{ACT.}}{V_{Th.}} = \frac{13.467}{14}$$

$= 0.961$

$$\boxed{C_V = 0.961}$$

$C_d = C_V \times C_c.$

$$C_c = \frac{0.642}{0.961} = 0.668$$

$$\boxed{C_c = 0.668}$$

3. A 25mm diameter nozzle discharges $0.76m^3$ of water per minute, when the head is 60m. The diameter of the jet is 22.5mm

Determine the value of (i) Coefficient of contraction, coefficient of velocity and coefficient of discharge.

(iii) The loss of head due to fluid resistance.

A pipe 100mm in diameter has a nozzle attached to it at the discharge end, the diameter of the nozzle is 50mm. The rate of discharge of water through the nozzle is 20 Litres/sec and the pressure at the base of the nozzle is $5.886\ N/cm^2$. Calculate the coefficient of discharge. Assume the base of the nozzle and outlet of the nozzle are at the same elevation.

$D_0 = 50mm.$

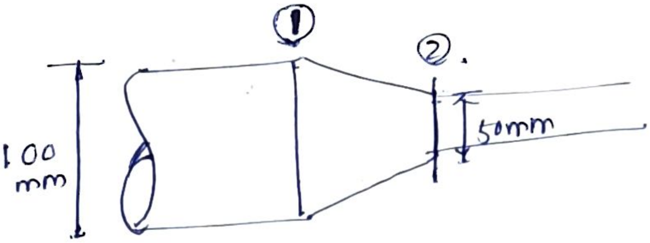
$Q = 20\ Ltr/sec.$

Act $= 0.02\ m^3/sec.$

$P = 5.886\ N/cm^2$

$= 5.886\times10^4\ N/m^2.$

$A_0 = \frac{\pi}{4}\times0.05^2 = 1.963\times10^{-3}\ m^2.$



$H = \frac{P}{\rho g} = \frac{5.886\times10^4}{1000\times9.81} = 6m.$

$V_{Th.} = \sqrt{2gH} = \sqrt{2\times9.81\times6}$

$= 10.849\ m/s.$

$\boxed{V_{Th.} = 10.85\ m/s.}$

$Q_{Th.} = 10.85\times1.963\times10^{-3}$

$= 0.0212\ m^3/sec.$

$Cd = \frac{0.02}{0.0212} = 0.943$

$\boxed{Cd = 0.943}$

From continuity equation:

$A_1V_1 = A_2V_2.$

$\left(\frac{\pi}{4}\times0.1^2\right)V_1 = \left(\frac{\pi}{4}\times0.05^2\right)\times V_2.$

$V_1 = 0.25V_2.$

$\boxed{V_1 = 0.25V_2.}$

Apply Bernoulli's Theorem b/w sec ① & ②:

$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2.$

$Z_1 = Z_2.$

$\frac{5.886\times10^4}{1000\times9.81} + \frac{(0.25V_2)^2}{2\times9.81} = 0 + \frac{V_2^2}{2\times9.81}$

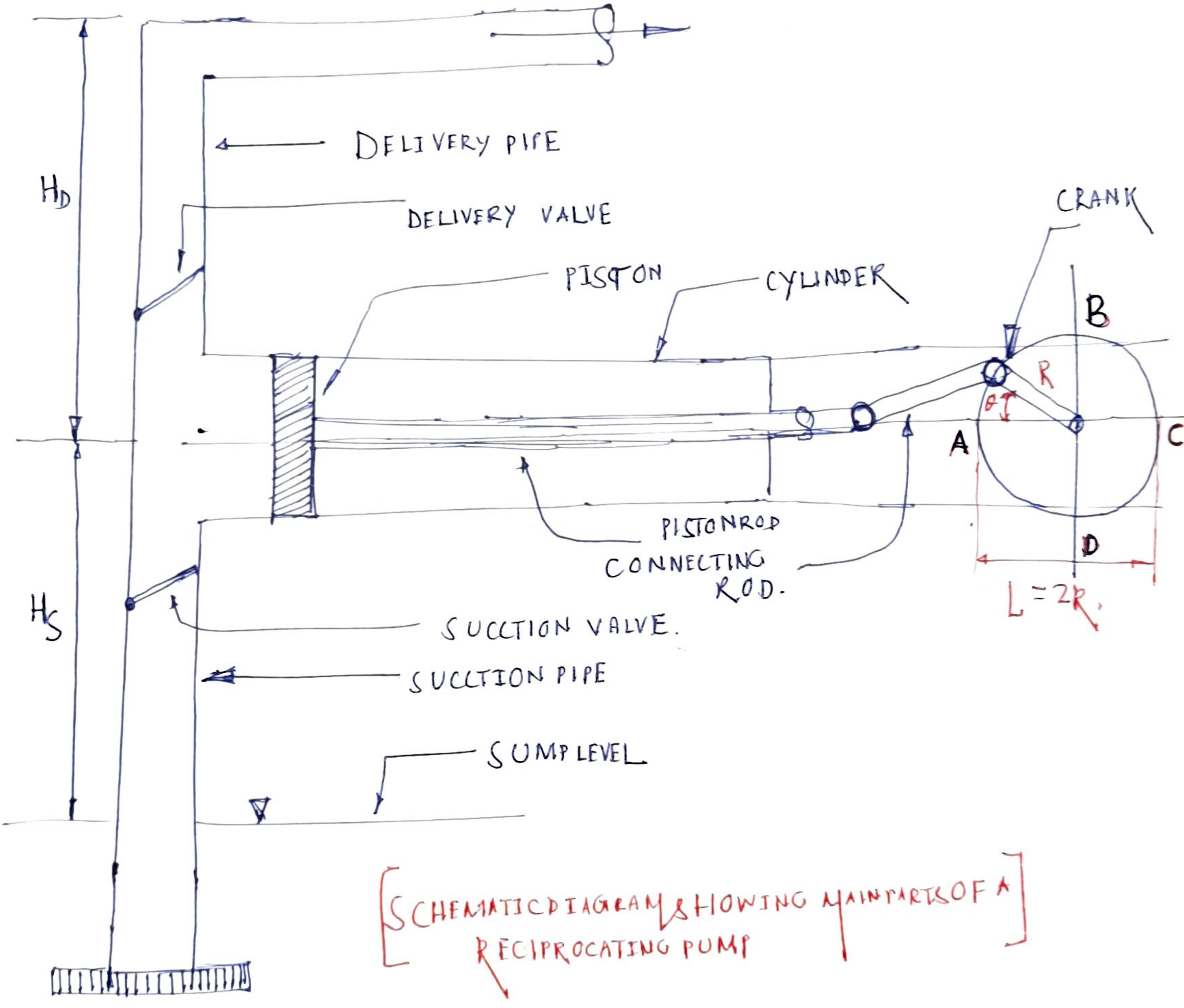
$\frac{5.886\times10^4}{1000\times9.81} = \frac{V_2^2}{2\times9.81}(1-0.25^2).$

$V_2 = \sqrt{\frac{5.886\times10^4}{1000\times9.81}\times\left(\frac{2\times9.81}{1-0.25^2}\right)}.$

RECIPROCATING PUMP

If the mechanical energy is converted into hydraulic energy, by means of centrifugal force acting on the liquid the pump is known as centrifugal pump.

If the mechanical energy is converted into hydraulic energy or pressure energy by sucking the liquid into a cylinder in which a piston is reciprocating (moving backwards and forwards) which exerts thrust on the liquid and increases the hydraulic energy or pressure energy, the pump is known as reciprocating pump.



[SCHEMATIC DIAGRAM SHOWING MAIN PARTS OF A RECIPROCATING PUMP]

— 3h ... $(\theta/2)\sqrt{2g}\ H^{5/2}$.

MAIN PARTS OF A RECIPROCATING PUMP

1. A cylinder with a piston, piston rod, connecting rod and a crank.
2. Suction pipe
3. Suction valve
4. Delivery pipe
5. Delivery valve.

WORKING PRINCIPLE OF A RECIPROCATING PUMP :-

1. The single acting reciprocating pump consists of a piston which moves forwards and backwards in a close fitting cylinder.

2. The movement of piston is obtained by connecting the piston rod to crank by means of connecting rod.

3. The crank is obtained by means of an electric motor.
Suction and delivery pipes with suction and delivery valves are connected to the cylinder.

4. The suction and delivery valves are one way valves or non return valves which allow the water to flow in one direction only.

5. Suction valve allows water from suction pipe to the cylinder which delivery valve allows water from cylinder to delivery pipe only.

6. When crank starts rotating, the piston moves to and fro in the cylinder.
When crank is at A the piston is at the extreme left position in the cylinder

7. As the crank is rotating from A to C i.e. $\theta = 0°$ to $\theta = 180°$, the piston is moving towards right in the cylinder.

8. The movement of piston towards right creates a partial vaccum in the cylinder. But on the surface of liquid in the sump, atmospheric pressure is acting, which is more than the pressure inside the cylinder.

9. The liquid is forced in the suction pipe from the sump, The liquid opens the suction valve and enters the cylinder.

10. When the crank is rotating from C to A i.e. 180° to 360°. The piston from its extreme right position starts moving towards left in the cylinder.

11. The movement of piston towards left increases the pressure of the liquid inside the cylinder more than atmospheric pressure.

12. Suction valve closes and delivery valve opens. The liquid is forced into the delivery pipe and is raised to a required height.

DISCHARGE THROUGH A RECIPROCATING PUMP:

Consider a single acting reciprocating pump: Means water is acting on one side of the piston only.

D = Diameter of the cylinder

A = Cross-sectional area of the piston or cylinder

$$\boxed{A = \frac{\pi}{4} \times D^2}$$

R = Radius of crank.

N = R.P.M of crank.

L = length of the stroke = 2R.

H_S = Height of axis of the cylinder from water surface in sump.

or

Suction height

or

Suction head.

H_D = Height of delivery outlet above the cylinder axis

or

delivery height

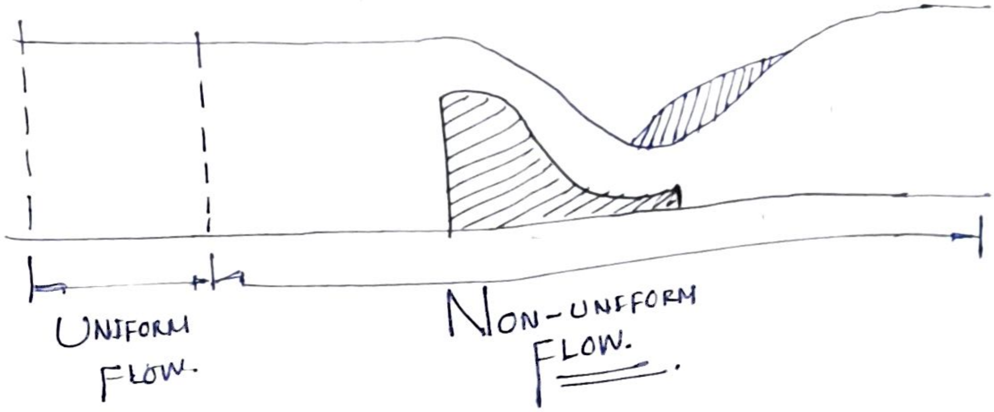
or

delivery head.

Volume of water delivered in one revolution or discharge of water in one revolution. = Area x Length of stroke

$V = A \times L$

$= A \times 2R = 2AR.$



GRADUALLY VARIED FLOW (G.V.F) :-

If the depth of flow in a channel changes gradually over a long length of channel, the flow is said to be gradually varied flow.

3. LAMINAR FLOW & TURBULENT FLOW :-

The flow in open channel is said to be laminar if the reynold's number Re is less than 500.

$$\boxed{Re = \frac{\rho V R}{\mu}}.$$

V = mean velocity of flow of water

R = Hydraulic radius or Hydraulic mean depth.

$$= \frac{\text{Cross-sectional area of flow normal to the direction of flow}}{\text{Wetted perimeter}}$$

$$\boxed{R = \frac{A}{P}}$$

If the Reynold number is more than 2000, the flow is said to be turbulent.

Re lies b/w 500-2000. The flow is considered to be in transition state.

If at any point in open channel flow, the velocity of flow, depth of flow or rate of flow changes w.r.t time is said to be unsteady flow.

Mathematically, $\frac{dv}{dt} \neq 0$, $\frac{dQ}{dt} = 0$, $\frac{dy}{dt} \neq 0$.

2. UNIFORM & NON UNIFORM FLOW :-

If for a given length of the channel, the velocity of flow, depth of flow, slope of the channel and cross-section remains constant, the flow is said to be uniform.

$$\frac{\partial v}{\partial s} = 0 , \quad \frac{\partial y}{\partial s} = 0$$

If for a given length of the channel, the velocity of flow, depth of flow do not remain constant, the flow is said to be non-uniform flow.

$$\frac{\partial v}{\partial s} \neq 0 , \quad \frac{dy}{ds} \neq 0$$

Non-uniform flow in open channels is also called varied flow.

Non-uniform flow ⟨ Rapidly varied flow (RVF) / gradually varied flow (GVF)

RAPIDLY VARIED FLOW :

Rapidly varied flow is defined as that flow in which depth of flow changes abruptly over a small length of channel.

When there is any obstruction in the path of flow of water, the level of water rises above the obstruction and then falls and again rises over a small length of channel.

The depth of flow changes rapidly over a short length of the channel. For this short length of channel the flow is called rapidly varied flow. (RVF).

FLOW IN OPEN CHANNELS

Flow in open channels is defined as the flow of a liquid with a free surface.

A free surface is a surface having constant pressure such as atmospheric pressure.

A liquid flowing at atmospheric pressure through a passage is known as flow in open channels.

(If the fluid is water, hence the flow of water through a passage under atmospheric pressure is called flow in open channels).

The flow of water through pipes at atmospheric pressure or when the level of water in the pipe is below the top of the pipe, is also called as open channel flow.

In case of open channel flow, as the pressure is atmospheric, the flow takes place under the force of gravity which means the flow takes place due to the slope of the bed of channel.

The hydraulic gradient line coincides with the free surface of water.

CLASSIFICATION OF FLOW IN CHANNELS:-

1. STEADY FLOW & UNSTEADY FLOW:-

The flow characteristics such as depth of flow, velocity of flow, rate of flow at any point in open channel flow do not change w.r.to time, the flow said to be steady.

Mathematically, $\frac{dv}{dt} = 0$, $\frac{dQ}{dt} = 0$, $\frac{dy}{dt} = 0$.

V = velocity of flow y = depth of flow.

Q = Rate of flow.

Number of revolution per second $= \frac{N}{60}$

Discharge of the pump per second

Q = Discharge in one revolution x No. of revolution per second.

$= AL\frac{N}{60}$

$$\boxed{Q = AL\frac{N}{60}}$$

Weight of water delivered per second ; $W = \rho g Q$.

$$\boxed{W = \rho g . AL\frac{N}{60}}$$

WORKDONE BY A RECIPROCATING PUMP :-

Workdone by the reciprocating pump per second

Workdone = Force x displacement

Workdone per second = Weight of water lifted per second X Total height through which water is lifted.

$$W.D. = W \times (H_S + H_D)$$

$$= \rho g AL\left(\frac{N}{60}\right)(H_S + H_D)$$

$$\boxed{W.D. = \rho g AL\left(\frac{N}{60}\right)(H_S + H_D)}$$

Power required to drive the pump (KW) : $\frac{W.D.}{1000} = \frac{\rho g AL\left(\frac{N}{60}\right)(H_S + H_D)}{1000}$.

4. SUB CRITICAL, CRITICAL, & SUPER CRITICAL FLOW:-

Froude number: $\boxed{F_e = \frac{V}{\sqrt{gD}}}$.

D = Hydraulic mean depth of channel.

$= \frac{\text{Wetted Area.}}{\text{Top width.}} = \frac{A}{T}.$

$$\boxed{D = \frac{A}{T}.}$$

$F_e < 1$: The flow is called sub-critical flow, tranquil or streaming flow.

$F_e = 1$: The flow is called critical flow.

$F_e > 1$: The flow is called Super-critical or shooting or rapid or torrential flow.

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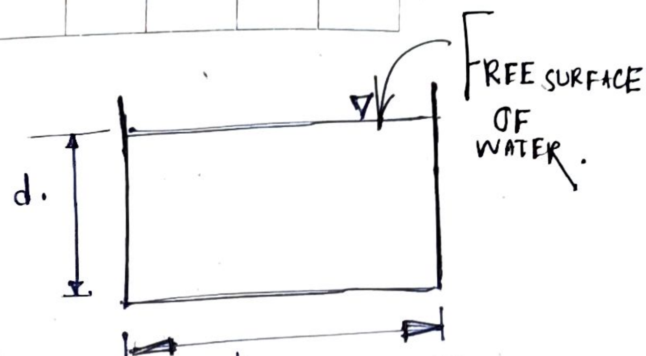
1. RECTANGULAR CHANNEL:



[CHANNEL OF RECTANGULAR C/S.]

(Area) $A = b \times d \Rightarrow \boxed{b = \frac{A}{d}}$

(Wetted perimeter) $P = b + 2d.$

$= \frac{A}{d} + 2d$

$$\boxed{P = \frac{A}{d} + 2d.}$$

For most economical section: (wetted perimeter) (P) should be minimum.

$$\frac{dP}{d(d)} = 0.$$

$$\frac{d}{d(d)}\left[\frac{A}{d} + 2d\right] = 0$$

$$-\frac{A}{d^2} + 2 = 0$$

$$A = 2d^2$$

$$b \times d = 2d^2$$

$$\boxed{b = 2d.}$$

$$m = \frac{A}{P} = \frac{b \times d}{b + 2d.}$$

$$= \frac{(2d) \times d}{2d + 2d.}$$

$$= \frac{2d^2}{4d.} = \frac{d}{2}.$$

$$\boxed{m = \frac{d}{2}}$$

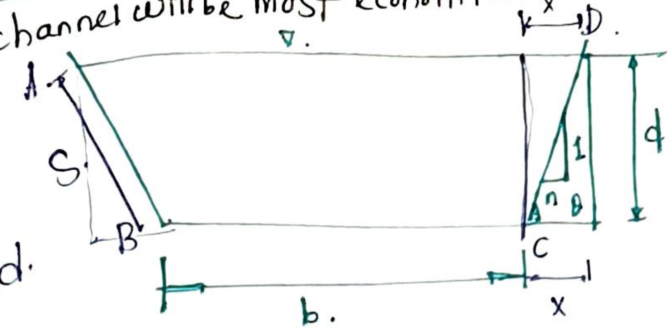
NOTE:- The rectangular section will be most economical when

(I) width is two times the depth of flow.

(II) Hydraulic mean depth is half the depth of flow.

2. MOST ECONOMICAL TRAPEZOIDAL CHANNEL:

The trapezoidal section of a channel will be most economical when its wetted perimeter is minimum.



$$A = \left(\frac{BC+AD}{2}\right) \times d$$

$$= \left[\frac{b+(b+2nd)}{2}\right] \times d.$$

$BC = b$

$AD = b+2nd.$

$\frac{1}{n} = \frac{d}{x}.$

$\boxed{x = nd.}$

$$= \left(\frac{2b+2nd}{2}\right) \times d$$

$$= (b+nd)d.$$

$$\boxed{A = bd+nd^2 \text{ or } (b+nd)d}$$

$\Rightarrow.\ \frac{A}{d} = b+nd.$

$$\boxed{b = \frac{A}{d} - nd.}$$

$$S = \sqrt{d^2+(nd)^2}$$

$$= \sqrt{d^2(1+n^2)}$$

$$= d\sqrt{(1+n^2)}$$

$$\boxed{S = d\sqrt{(1+n^2)}}$$

$$P = b+2S.$$

$$\boxed{P = b+2d\sqrt{1+n^2}.}$$

$$= \left(\frac{A}{d} - nd\right) + 2d\sqrt{1+n^2}$$

$$\boxed{P = \left(\frac{A}{d} - nd\right) + 2d\sqrt{1+n^2}}$$

The side slope is given as 1 vertical to n horizontal.

(1). For most economical section: P should be minimum.

$$\frac{dP}{d(d)} = 0$$

$$\Rightarrow \frac{d}{d(d)}\left[\left(\frac{A}{d} - nd\right) + 2d\sqrt{1+n^2}\right] = 0$$

$$\Rightarrow -\frac{A}{d^2} - n + 2\sqrt{1+n^2} = 0.$$

$$\Rightarrow.\ 2\sqrt{1+n^2} - n = \frac{A}{d^2}$$

$$2\sqrt{1+n^2} - n = \frac{(b+nd)d}{d^2}. \qquad \because A = (b+nd)d.$$

$$= \frac{b+nd}{d}.$$

$$2\sqrt{1+n^2} = \frac{b+nd}{d} + n.$$

$$= \frac{b+nd+nd}{d}.$$

$$= \frac{b+2nd}{d}.$$

$$2d\sqrt{1+n^2} = b+2nd.$$

$$\boxed{2d\sqrt{1+n^2} = b+2nd.}$$

NOTE:

$$\frac{b+2nd}{2} = d\sqrt{1+n^2} = S$$

Topwidth.

Half of ~~width~~ = one of the slopping side

(2). Hydraulic Meandepth: $m = \frac{A}{P}$

$A = (b+nd)d.$

$P = b+2d\sqrt{1+n^2}$

$$= \frac{(b+nd)d}{b+(2d\sqrt{1+n^2})}.$$

$$= \frac{(b+nd)d}{b+(b+2nd)} \qquad \because 2d\sqrt{1+n^2} = b+2nd.$$

$$= \frac{(b+nd)\times d.}{2(b+nd)}$$

$$= \frac{d}{2}.$$

$$\boxed{m = \frac{d}{2}}$$

NOTE: For a trapezoidal section to be most economical by hydraulic meandepth must be equal to half of depth of flow.

$H^{5/2}$.

$$A = R^2\theta - \frac{R^2 \sin 2\theta}{2}$$

$$= R^2\left(\theta - \frac{\sin 2\theta}{2}\right).$$

$$\boxed{A = R^2\left[\theta - \frac{\sin 2\theta}{2}\right]}$$

Hydraulic mean depth : $m = \frac{A}{P}$.

$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$

$P = 2R\theta$

$$= \frac{R^2\left(\theta - \frac{\sin 2\theta}{2}\right)}{2R\theta}$$

$$= \frac{R\left(\theta - \frac{\sin 2\theta}{2}\right)}{2\theta}.$$

$$\boxed{m = \frac{R}{2\theta}\left(\theta - \frac{\sin 2\theta}{2}\right)}$$

Discharge : $\boxed{Q = AC\sqrt{mi}}$.

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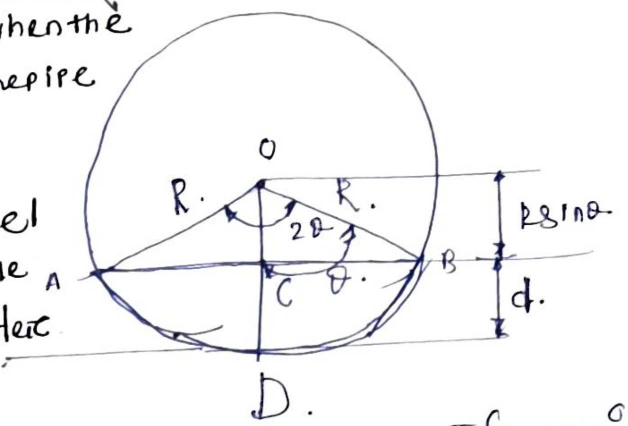
MOST ECONOMICAL CIRCULAR SECTION

The flow of liquid through a circular pipe, when the level of liquid in the pipe is below the top of the pipe is classified as an open channel flow.

The Rate of flow through circular channel is determined from the depth of flow and angle subtended by the liquid surface at the center of the circular channel.



Wetted Perimeter $P = \frac{2\pi R}{2\pi} \times 2\theta$

$= 2\theta R.$

$\boxed{P = 2R\theta}$

$\pi^c = 180^\circ$

$2\pi R = 2\pi. = 2 \times 180^\circ$

$= 360^\circ.$

$2\pi R = 360^\circ = 2\pi.$

$\frac{2\pi R}{2\pi} = \frac{X}{2\theta}.$

$\boxed{X = \frac{2\pi R}{2\pi} \times 2\theta}$

Wetted Area: A = Area of sector OADBO − △ABO.

Area of sector OADBO $= \frac{\pi R^2}{2\pi} \times 2\theta.$

$= R^2\theta.$

$\pi R^2 \to 2\pi.$

$A \to 2\theta.$

$\frac{\pi R^2}{2\pi} = \frac{X}{2\theta}$

$\boxed{A = \frac{\pi R^2}{2\pi} \times 2\theta.}$

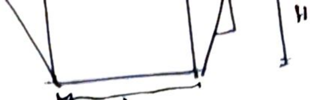
$\triangle ABO = \frac{AB \times OC}{2}.$

$OC = R\sin\theta.$
$AB = 2BC$
$= 2R\cos\theta.$

$= \frac{2R\cos\theta \times R\sin\theta}{2}.$

$= R^2 \sin\theta.\cos\theta$

$= \frac{R^2 \sin 2\theta}{2}.$



$\ldots (b/a)\sqrt{2g}\, H^{5/2}.$

BEST SIDE SLOPE FOR MOST ECONOMICAL TRAPEZOIDAL SECTION :-

$A = (b + nd)d. \Rightarrow \boxed{b = \frac{A}{d} - nd.}$

$P = b + 2d\sqrt{1+n^2}$

$= \left(\frac{A}{d} - nd\right) + 2d\sqrt{1+n^2}$

$$\boxed{P = \left(\frac{A}{d} - nd\right) + 2d\sqrt{1+n^2}}$$

b = width of the channel
d = depth of the flow
A = Area of Trapezoidal Section
P = wetted perimeter.

Best side slope will be when the section is most economical section or P to be minimum.

depth of flow and area are constant.

$\frac{dP}{dn} = 0$

$\frac{d}{dn}\left[\left(\frac{A}{d} - nd\right) + 2d\sqrt{1+n^2}\right] = 0$

n = slope of the side of the channel.

$0 - d + 2d \times \frac{1}{2}(1+n^2)^{1/2-1} \times 2n = 0$

$-d + d(1+n^2)^{-1/2} \times 2n = 0$

$-d + \frac{2nd}{\sqrt{1+n^2}} = 0.$

$\frac{2nd}{\sqrt{1+n^2}} = d.$

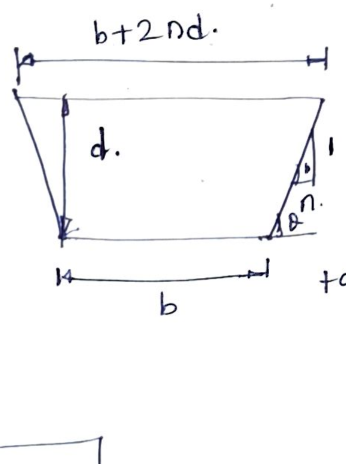
$2nd = d\sqrt{1+n^2}$

$2n = \sqrt{1+n^2}$

$4n^2 = 1 + n^2$

$3n^2 = 1.$

$n = \frac{1}{\sqrt{3}}$



$\tan\theta = \frac{1}{n}.$

$\frac{1}{n} = \tan\theta = \frac{1}{1/\sqrt{3}}.$

$\tan\theta = \tan 60^\circ \Rightarrow \boxed{\theta = 60^\circ}$

Half of Top width = Length of one Sloping side.

$$\frac{b+2nd}{2} = d\sqrt{1+n^2}$$

$$n = 1/\sqrt{3}$$

$$\frac{b+\frac{2d}{\sqrt{3}}}{2} = d\sqrt{1+\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$b+\frac{2d}{\sqrt{3}} = 2d\sqrt{1+\frac{1}{3}}$$

$$= 2d\sqrt{\frac{4}{3}}$$

$$= 2d \cdot \frac{2}{\sqrt{3}}$$

$$b+\frac{2d}{\sqrt{3}} = \frac{4d}{\sqrt{3}}.$$

$$b = \frac{4d}{\sqrt{3}} - \frac{2d}{\sqrt{3}} = \frac{2d}{\sqrt{3}}.$$

$$\boxed{b = \frac{2d}{\sqrt{3}}}$$

$$P = b+2d\sqrt{1+n^2}$$

$$= \frac{2d}{\sqrt{3}} + 2\times d\sqrt{1+\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{2d}{\sqrt{3}} + 2d\sqrt{1+\frac{1}{3}}$$

$$= \frac{2d}{\sqrt{3}} + 2d \times \frac{2}{\sqrt{3}}.$$

$$= \frac{2d}{\sqrt{3}} + \frac{4d}{\sqrt{3}} = \frac{6d}{\sqrt{3}} = 3\left(\frac{2d}{\sqrt{3}}\right) = 3b.$$

$$\boxed{P = 3b}$$

NOTE: (I) Hence best side slope is at 60° to the horizontal.

(II). For a slope of 60° the length of the sloping side is equal to the width of the trapezoidal Section.

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$$Q = AC\sqrt{mi}$$

$$= AC\sqrt{\frac{A}{P}i}$$

$$= C\sqrt{\frac{A^3 i}{P}}$$

$$\frac{d}{d\theta}\left(\frac{A^3}{P}\right) = 0.$$

$$\frac{P.\frac{dA}{d\theta}.3A^2. - A^3.\frac{dP}{d\theta}}{P2} = 0.$$

$$3A^2P.\frac{dA}{d\theta} - A^3.\frac{dP}{d\theta} = 0.$$

$$3A^2P\frac{dA}{d\theta} = A^3\frac{dP}{d\theta}$$

$$3P\frac{dA}{d\theta} = A.\frac{dP}{d\theta}.$$

$$P = 2R\theta$$

$$\frac{dP}{d\theta} = 2R.$$

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$$

$$\frac{dA}{d\theta} = R^2\left(1 - \frac{\cos 2\theta \times 2}{2}\right).$$

$$= R^2(1-\cos 2\theta).$$

$$3 \times 2R\theta \times R^2(1-\cos 2\theta) = R^2\left(\theta - \frac{\sin 2\theta}{2}\right) \times 2R$$

$$3\theta \times 2R^3(1-\cos 2\theta) = 2R^3\left(\theta - \frac{\sin 2\theta}{2}\right).$$

$$3\theta - 3\theta\cos(2\theta) = 2\theta - \frac{\sin 2\theta}{2}.$$

$$6\theta - 6\theta\cos 2\theta = 2\theta - \sin 2\theta$$

$$6\theta - 2\theta = 6\theta\cos 2\theta - \sin 2\theta$$

$$4\theta = 6\theta\cos 2\theta - \sin 2\theta$$

$$\boxed{\begin{aligned} 2\theta &= 308^\circ \\ \theta &= 154^\circ \end{aligned}}$$

$$d = R - R\cos\theta = R(1-\cos\theta).$$

$$= R\left[1 - \cos(154^\circ)\right]$$

$$= 1.9R.$$

$$= 0.95D.$$

$$\boxed{\begin{aligned} d &= 1.90R \\ &\text{OR} \\ &0.95D. \end{aligned}}$$

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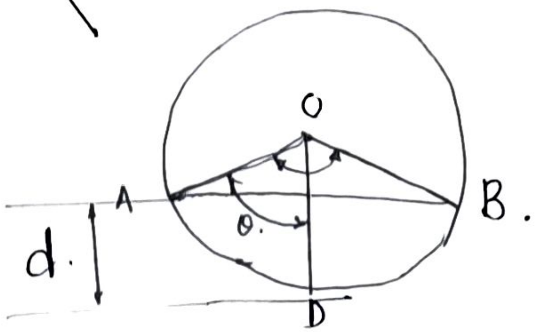
CONDITION FOR MAXIMUM VELOCITY FOR CIRCULAR SECTION:-



$$m = \frac{A}{P} \qquad A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$$

$$P = 2R\theta.$$

$$\Rightarrow \frac{dm}{d\theta} = 0.$$

$$\Rightarrow \frac{d\left(\frac{A}{P}\right)}{d\theta} = 0.$$

$$\Rightarrow \frac{d}{d\theta}\left(\frac{A}{P}\right) = 0$$

$$\Rightarrow \frac{P\cdot\frac{dA}{d\theta} - A\cdot\frac{dP}{d\theta}}{P^2} = 0.$$

$$\Rightarrow \frac{2R\theta \times \frac{d}{d\theta}\left[R^2\left(\theta - \frac{\sin 2\theta}{2}\right)\right] - R^2\left(\theta - \frac{\sin 2\theta}{2}\right) \times \frac{d}{d\theta}(2R\theta)}{(2R\theta)^2} = 0.$$

$$\Rightarrow 2\theta R \times \frac{d}{d\theta}\left(R^2\theta - \frac{\sin 2\theta}{2}R^2\right) = R^2\left(\theta - \frac{\sin 2\theta}{2}\right) \times \frac{d}{d\theta}(2R\theta)$$

$$\Rightarrow 2\theta R \times \left[R^2 - \frac{\cos 2\theta \cdot 2 \cdot R^2}{2}\right] = R^2\left(\theta - \frac{\sin 2\theta}{2}\right) \times 2R.$$

$$\Rightarrow 2\theta R \cdot R^2\left[1 - \cos 2\theta\right] = 2R^3\left(\theta - \frac{\sin 2\theta}{2}\right).$$

$\sqrt{2g}\ H^{5/2}$.

$$\theta(1-\cos 2\theta)\, 2R^3 = 2R^3\left(\theta - \frac{\sin 2\theta}{2}\right).$$

$$\theta - \theta\cos 2\theta = \theta - \frac{\sin 2\theta}{2}.$$

$$\theta\cos 2\theta = \frac{\sin 2\theta}{2}.$$

$$\tan 2\theta = 2\theta.$$

$$\boxed{\begin{gathered} 2\theta = 257^\circ 30' \\ \theta = 128^\circ 45' \end{gathered}}$$

$$d = OD - OC$$

$$= R - R\cos\theta = R(1-\cos\theta).$$

$$= R\left[1-\cos(128^\circ 45')\right]$$

$$= R\left[1-(-0.625)\right]$$

$$= R(1+0.625) = 1.625R.$$

$$\boxed{d = 1.625R}$$

$$\boxed{\begin{gathered} d = 1.625R \\ \text{OR} \\ 0.8125D. \end{gathered}}$$

$$m = \frac{R}{2\theta}\left(\theta - \frac{\sin 2\theta}{2}\right)$$

$$= \frac{R}{2\times 128^\circ 45'}\left(128^\circ 45' - \frac{\sin(2\times 128^\circ 45')}{2}\right).$$

$$= 0.6R = 0.3D.$$

$$\boxed{\begin{gathered} m = 0.6R \\ \text{OR} \\ 0.3D \end{gathered}}$$

Topwidth : $b+2X$

$= 8+2\times0.8$

$= 8+1.6 = 9.6\,m.$

Area $= \frac{1}{2}\left(\text{Topwidth}+\text{bottomwidth}\right)\times \text{Total depth.}$

$= \left(\frac{8+9.6}{2}\right)\times 2.4.$

$= 17.6\times1.2 = 21.12\,m^2$

$\boxed{A = 21.12\,m^2}$

$S = \sqrt{0.8^2+2.4^2}$

$= 2.529\,m.$

$\boxed{S = 2.529\,m.}$

$P = b+2S = 8+2\times2.529 = 13.058\,m.$

$m = \frac{A}{P} = \frac{21.12}{13.058}$

$= 1.617\,m$

$\boxed{m = 1.617\,m.}$

$V = C\sqrt{mi}$

$= 50\sqrt{1.617\times\left(\frac{1}{4000}\right)}$

$= 1.005\,m/s.$

$Q = AV = 21.12\times1.005 = 21.225\,m^3/sec.$

$$Q = AC\sqrt{mi}$$

$$i = \left(\frac{Q}{AC\sqrt{m}}\right)^2$$

$$= \left(\frac{0.1}{0.18 \times 56\sqrt{0.15}}\right)^2$$

$$= \frac{1}{\left(\frac{0.18 \times 56 \times \sqrt{0.15}}{0.1}\right)^2}$$

$$= \frac{1}{1524}$$

$$\boxed{i = \frac{1}{1524}}$$

$$Q = AC\sqrt{mi}$$

$$= (AC\sqrt{m})\sqrt{i}$$

$$= K\sqrt{i}$$

$K =$ conveyance of the channel $= 0.18 \times 56 \times \sqrt{0.15}$

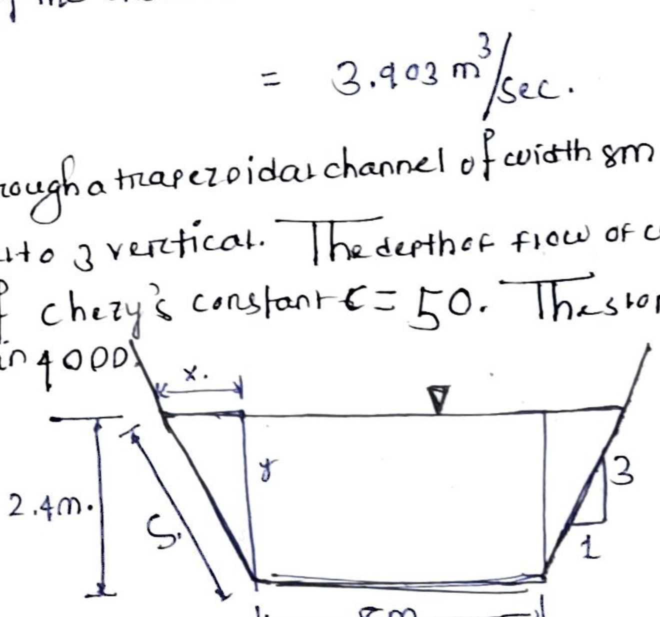
$= 3.903\ m^3/sec.$

4. Find the discharge through a trapezoidal channel of width 8m and side slope of 1 horizontal to 3 vertical. The depth of flow of water is 2.4m and value of chezy's constant $C = 50$. The slope of the bed of the channel is given 1 in 4000.

$C = 50.$

$i = \frac{1}{4000}.$

$\frac{x}{y} = \frac{1}{3}.$



$x = \frac{1}{3} \times 2.4 = 0.8m.$

$S = \sqrt{x^2+y^2}$

$m = \frac{A}{P} = \frac{10}{9}$

$= 1.11\ m.$

$V = C\sqrt{mi}$

$Q = AC\sqrt{mi}$

$20 = 10 \times 50 \times \sqrt{1.11 \times i}$

$\frac{20}{500 \times \sqrt{1.11}} = \sqrt{i}$

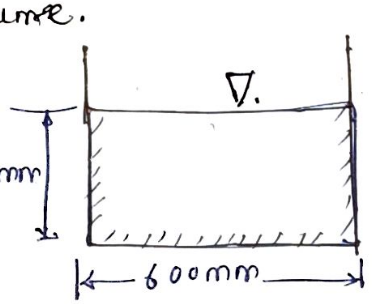
$i = \left(\frac{20}{500 \times \sqrt{1.11}}\right)^2 = 1.441 \times 10^{-3}$

$= \frac{1.441}{1000}$

$= \frac{1}{\left(\frac{1000}{1.441}\right)} = \frac{1}{694.}$

$\boxed{i = \frac{1}{694.}}$

3. A Flow of water of 100 Litres per second flows down in a rectangular flume of width 600mm and having adjustable bottom slope. If chezy's constant C is 56. Find the bottom slope necessary for uniform flow with a depth of flow of 300mm. Also Find the conveyance K of the flume.



$Q = 100\ Ltr/second$

$= 0.1\ m^3/sec.$

$C = 56$

$b = 600mm = 0.6\ m$

$d = 300mm = 0.3\ m$

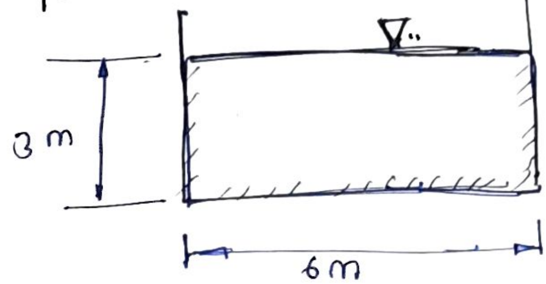
$A = b \times d = 0.6 \times 0.3 = 0.18\ m^2$

$P = b + 2d = 0.6 + 0.3 \times 2 = 1.20\ m$

$M = \frac{A}{P} = \frac{0.18}{1.2}$

$= 0.15\ m.$

1. Find the velocity of flow and rate of flow through a rectangular channel of 6m wide, 3m deep, when it is running full. The channel is having bed slope as 1 in 2000. Take chezy's constant as $C = 55$.



$b = 6\,m$

$d = 3\,m.$

$i = \frac{1}{2000}$

$C = 55$

$A = b \times d = 6 \times 3 = 18 m^2$

$m = \frac{A}{P} = \frac{b \times d}{b + 2d} = \frac{18}{6 + 2 \times 3} = \frac{18}{12} = \frac{3}{2} = 1.5\,m.$

$V = C\sqrt{mi}$

$= 55 \times \sqrt{1.5 \times \left(\frac{1}{2000}\right)}$

$= 1.506\,m/s.$

$\boxed{V = 1.506\,m/s.}$

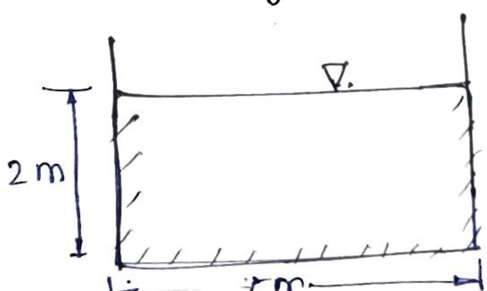
$Q = AV$

$= 18 \times 1.506$

$= 27.108\,m^3/sec.$

$\boxed{Q = 27.108\,m^3/sec.}$

2. Find the slope of the bed of a rectangular channel of width 5m when depth of water is 2m and rate of flow is given as 20 m^3/sec. Take chezy's constant $C = 50$.



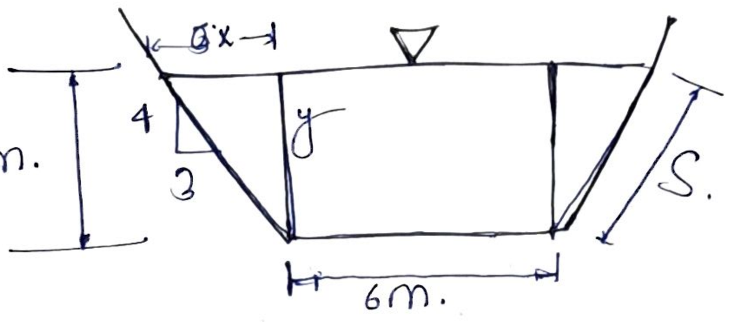
$b = 5\,m$

$d = 2\,m$

$A = b \times d = 5 \times 2 = 10\,m^2$

$P = b + 2d = 5 + 2 \times 2 = 9\,m.$

5. Find the bed slope of trapezoidal channel of bed width 6m, depth of water 3m and side slope of 3 horizontal to 4 vertical. When the discharge through the channel is $30\ m^3/sec$. Take chezy's constant $C = 70$.



$Q = 30 m^3/sec$

$C = 70.$

$\frac{x}{y} = \frac{3}{4}.$

$x = \frac{3}{4} \times 3$

$= \frac{9}{4} = 2.25m.$

bottom width = 6 m.

Top width = $2 \times 2.25 + 6 = 10.5m.$

$A = \left(\frac{10.5+6}{2}\right) \times 3 = 24.75 m^2.$

$S = \sqrt{2.25^2 + 3^2}$

$= 3.75m.$

$P = b + 2S$

$= 6 + 2 \times 3.75 = 13.5m.$

$m = \frac{24.75}{13.5}$

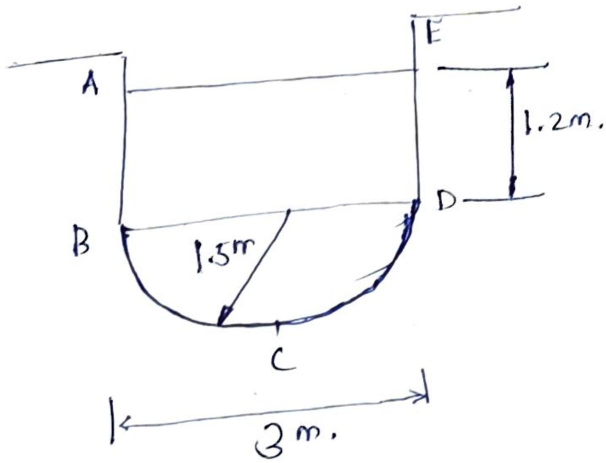
$= 1.833\ m$

$Q = AC\sqrt{mi}$

$i = \left(\frac{Q}{AC\sqrt{m}}\right)^2 = \frac{1}{\left(\frac{AC\sqrt{m}}{Q}\right)^2} = \frac{1}{\left(\frac{24.75 \times 70 \times \sqrt{1.833}}{30}\right)^2}$

$= \frac{1}{6113}$

6. Find the discharge of water through the channel shown below. Take the value of chezy's constant $C = 60$ and slope of the bed as 1 in 2000.



$C = 60$

$i = \frac{1}{2000}$.

Total Area.

$A =$ Area of ABDE + Area of BCD.

$= 1.2 \times 3 + \frac{\pi \times (1.5^2)}{2}$

$= 7.134 m^2$

$$\boxed{A = 7.134 m^2}$$

Wetted perimeter: $p = 2 \times 1.2 + \pi \times 1.5$

$= 7.112 m$

$$\boxed{p = 7.112 m}$$

Hydraulic meandepth: $m = \frac{A}{p}$

$= \frac{7.134}{7.112}$

$= 1.003$ m.

$$\boxed{m = 1.003 m.}$$

$Q = AC\sqrt{mi}$

$= 7.134 \times 60 \times \sqrt{1.003 \times \left(\frac{1}{2000}\right)}$

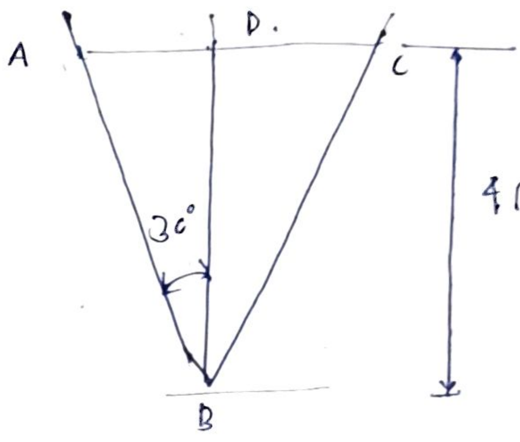
$= 9.585 m^3/sec$

$$\boxed{Q = 9.585 m^3/sec.}$$

7. Find the rate of flow of water through a ~~open~~ V-shaped open channel as shown below. Take $C = 55$ and slope of the bed 1 in 2000.

$C = 55$

$i = \frac{1}{2000}$.



$\tan(30°) = \frac{AD}{BD}$.

$AD = 4 \times \tan(30°)$

$= 2.309\ m$.

$AC = 2 \times 2.309 = 4.618\ m$.

Area of the triangle $A = \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times 4.618 \times 4$

$= 9.236\ m^2$

$$\boxed{A = 9.236\ m^2}$$

$p = AB + BC$

$= 2\ AB$

$= 2 \times 4.618$

$= 9.236\ m$

$$\boxed{p = 9.236\ m}$$

$AB = BC$

$AB = \sqrt{AD^2 + BD^2}$

$= \sqrt{2.309^2 + 4^2}$

$= 4.618\ m$.

$$\boxed{AB = 4.618\ m}$$

$m = \frac{9.236}{9.236} = 1\ m$.

$$Q = AC\sqrt{mi}$$

$$= 9.236 \times 55 \times \sqrt{1 \times \left(\frac{1}{2000}\right)}$$

$$= 11.358\, m^3/sec$$

$$\boxed{Q = 11.358\, m^3/sec.}$$